

Studies on Intensity Distribution of Point Spread Function of three zone filter by Complex Pupil Function

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Abstract:

In this paper we report the improvement of the resolution of an optical system in the presence of defocus with complex pupil function by introducing three level asymmetric filters of Hanning, Gaussian, and co-sinusoidal. Three zone aperture shading with amplitude and phase with all possible combinations in each zone have been tested for better results. Hanning amplitude complex pupil function under last zone of defocusing with $(-\pi/4)$ and $(\pi/3)$ phase filter situation provides the optical system to achieve super-resolver state. The highest degree of the amplitude anodization improves the lateral resolution of the central maxima.

Keywords: Amplitude and phase point spread function, Hanning function, Gaussian function, Co-sinusoidal function and three-point resolution.

1.1 Introduction

Among the mechanisms available nowadays to study an optical imaging system, the point spread function (PSF) takes an important role. In this approach, the object is decomposed into infinitesimal point sources and the image changes as the superposition of the field distribution corresponding to each point-source object changes. In order to improve the resolution of the optical system of three zone aperture, asymmetric apodization was introduced, the results had sharp central peak and with the help of different filters and suitable combinations of filters, reduced the side-lobes. Suitable combinations of different filters have been examined in terms of the reduction of secondary side-lobes by modifying the three zone aperture with different degrees of amplitude apodization β with defocus. The present research provides a significant contribution to study the resolution of PSF. Depending on the investigations done in the three zone apodization process, it can be inferred that the Co-sinusoidal amplitude filter in the first zone, Gaussian amplitude filter in the middle zone and Hanning amplitude filter in the outer zone could be the solution for modifying the point spread function of the optical system under the influence of defect-of-focus. In the present study, we studied the three zone aperture with the three orders which are Hanning, Gaussian, and co-sinusoidal amplitude masks with $(-\pi/4)$ and $(\pi/3)$ phase filters, to change the light intensity distribution of the optical system with asymmetric complex pupil filters.

1.2 Theory:

The aim of this study was to examine the effect of three zone filter with different combinations on the optical system. The Hanning amplitude mask is placed in the outer zone and the other two

filters are interchanged, the best outcome was for the combination of Co-sinusoidal amplitude filter in the first zone and Gaussian amplitude filter in the middle zones with $(-\pi/4)$ and $(\pi/3)$ phase filter. The general expression for diffraction field of two amplitude filters is given by:

$$S(\phi_d, Z) = 2 \left[\int_0^a f_1(x) J_0(Zx) x dx + \int_a^b f_2(x) J_0(Zx) x dx + \int_b^1 f_3(x) J_0(Zx) x dx \right] \dots \dots (1)$$

Here $f_1(x)$ is Co-sinusoidal, $f_2(x)$ is Gaussian, and $f_3(x)$ is Hanning mask pupil functions for the amplitude apodization of the pupil transmission. Z is the reduced dimensionless diffraction coordinate in the image plane.

The generalized expression for the amplitude and phase impulse response of the pupil function in the presence of three zone aperture:

$$S(\phi_d, Z) = \int_0^a 2 \left[f_1(x) e^{-\phi_d} J_0(Zx) x dx - (\pi/4) \int_a^b f_2(x) e^{-\phi_d} J_0(Zx) x dx + \left(\frac{\pi}{3}\right) \int_b^1 f_3(x) e^{-\phi_d} J_0(Zx) x dx \right] \dots \dots (2)$$

$$\text{Hanning-}f_1 = \cos(\pi\beta x) \dots \dots (3)$$

$$\text{Gaussian-}f_2 = \exp(-\beta r/2x^2) \dots \dots (4)$$

$$\text{CO-sinusoidal-}f_3 = 1 + \beta \cos(\pi r^2)/1 + \beta \dots \dots (5)$$

In which $J_0(Zx)$ is the zero order Bessel function of the first kind and zero order, β is the apodization coefficient, this controls the degree of apodization.

1.3 Results:

The results of investigations on the effects of asymmetric apodization on intensity distribution of complex pupil function in the image plane of an optical system have been obtained from Eq. (2) as a function of diffraction coordinate Z varying from -15 to $+15$ with three zone aperture by using Mat lab programme to study the effect of three filters with suitable order with apodization parameter β and ϕ_d variations with all combinations. The apodization parameter β varies from 0 to 1 in steps of 0.25. With $\beta = 0$ the optical system is said to be unapodized optical system. $\beta = 0$, corresponds to the Airy case (perfect lens) and for the values of $\beta \neq 0$ represents the apodised system. The influence of defect-of-focus (ϕ_d) on the optical system also varies from 0 to 2π with $\pi/2$ variations. Here $\phi_d = 0$ is the focus system and $\phi_d \neq 0$ gives the defocused optical system.

As the value of defect-of-focus (ϕ_d) ($=3\pi/2$) and as well as degree of apodization β ($=1$) is increasing the intensity that is the peak of the central maxima attains highest value and sharp peak with decrease of side lobes have been studied.

F1F3F2-combination

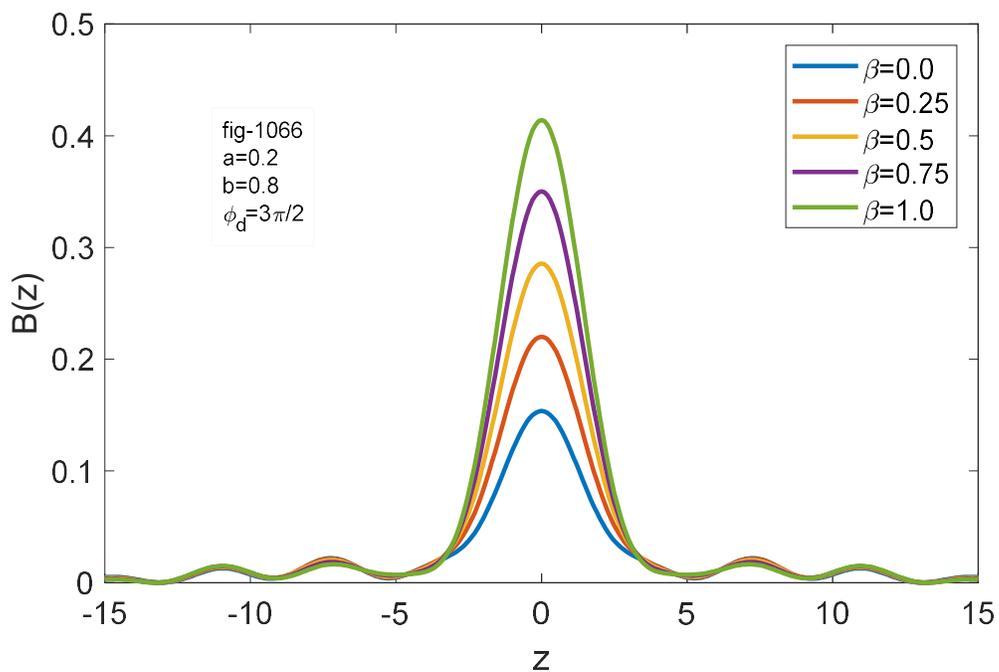
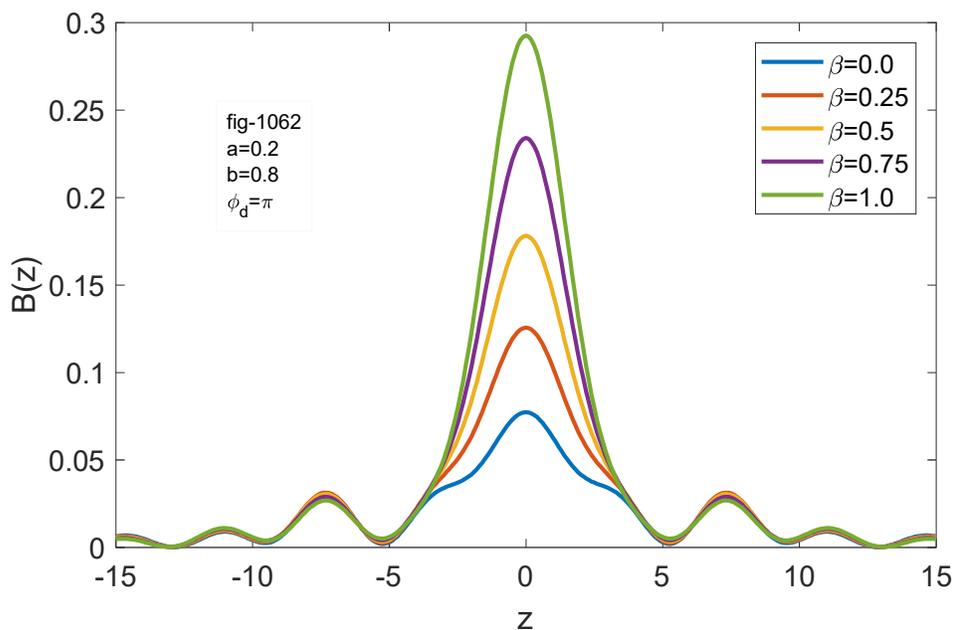


Fig.(1-2):Variation in the axial shape of the point spread function of three zone aperture for different degrees of Hanning filter(first zone), Co-sinusoidal filter(second zone) and Gaussian filter in the outer zone apodization varying values of ϕ_d is $\pi/2$, and $3\pi/2$.

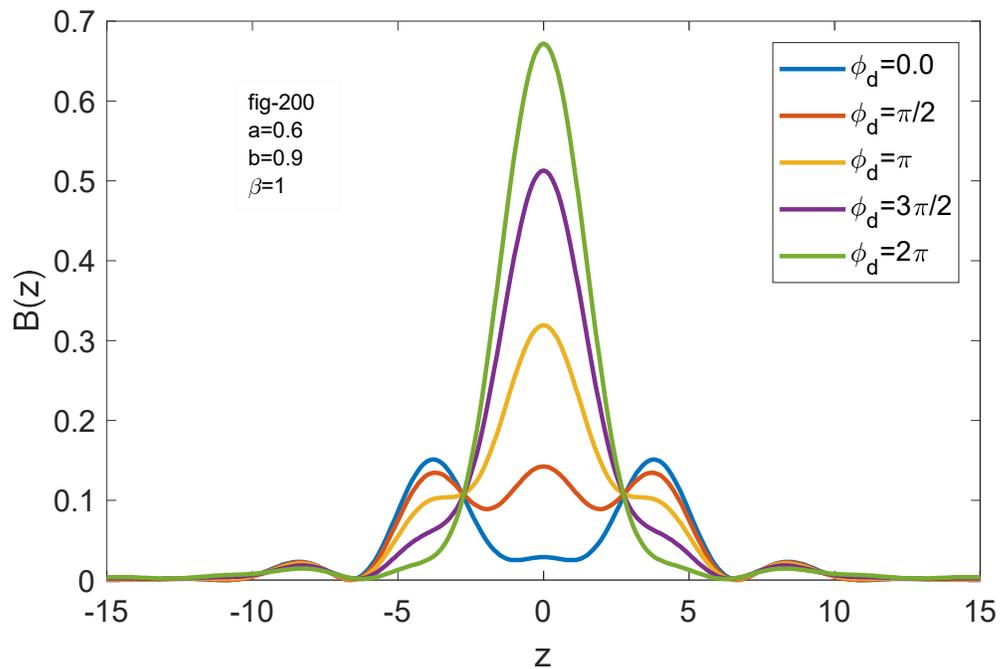
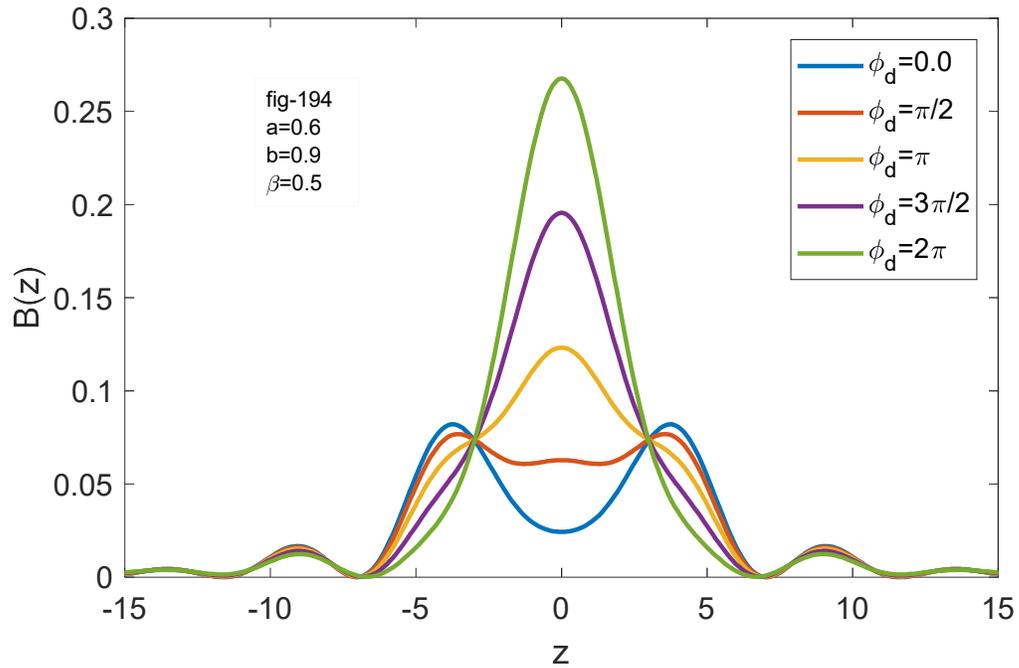
F3F2F1 combination:

Fig.(1-2): Variation in the axial shape of the point spread function of three zone aperture of variable ϕ_d values for different degrees of Co-sinusoidal filter -first zone, Gaussian filter - second zone and Hanningfilter in the outer zone apodization varying values of β is 0.5 and 1

		c. max		f. min		f.max		s.min		s.max	
a=0.2	0	0	0.1021	5.2404	0.002	7.3034	0.0309	9.5498	0.0025	11.0122	0.009
b=0.8	0.2	0	0.1156	5.2573	0.0024	7.3201	0.0314	9.571	0.003	11.024	0.0094
$\varnothing d = \pi$	0.4	0	0.1567	5.267	0.003	7.3273	0.031	9.5702	0.0034	11.0298	0.0098
1062	0.6	0	0.2001	5.2725	0.0036	7.328	0.03	9.555	0.0037	11.0325	0.0102
	0.8	0	0.2455	5.275	0.0043	7.324	0.0285	9.5298	0.0039	11.0339	0.0106
	1	0	0.2934	5.2749	0.0051	7.3157	0.0268	9.4972	0.0039	11.0346	0.011

F1F3F2	0	0	0.1537	5.252	0.004	7.2356	0.0219	9.2707	0.0035	10.9413	0.0129
a=0.2	0.2			5.2354	0.0045	7.2374	0.0216	9.2624	0.004	10.9488	0.0135
b=0.8	0.4	0	0.2596	5.213	0.0051	7.2279	0.0208	9.2374	0.0043	10.9533	0.0141
$\varnothing d = 3\pi/2$	0.6	0	0.3116	5.1787	0.0058	7.2089	0.0195	9.2019	0.0043	10.9567	0.0145
1066	0.8	0	0.3632	5.1173	0.0065	7.1801	0.0181	9.1592	0.0042	10.9597	0.0148
	1	0	0.4142	4.9549	0.0072	7.1392	0.0165	9.1115	0.004	10.9627	0.015

Maxima minima position and values of the point spread function of three zone aperture for different degrees of Hanning filter(first zone), Co-sinusoidal filter(second zone) and Gaussian filter in the outer zone apodization varying values of $\varnothing d$ is $\pi/2$, and $3\pi/2$.

a=0.6	0	0	0.0287	0.9497	0.025	3.786	0.1511	6.6539	6.00E-04	8.3935	0.023
b=0.9	$\pi/2$	0	0.1424	1.9446	0.089	3.7274	0.1346	6.6104	0	8.3651	0.0222
$\beta=1$	π	0	0.3192	6.5385	7.00E-04	8.3243	0.0208	10.9052	0.0015	11.9447	0.0022
fig-200	$3\pi/2$	0	0.5131	6.4246	0.0016	8.2835	0.0184				
	2π	0	0.6718	6.2456	0.0014	8.2753	0.0146				

		c. max		f. min		f.max		s.min		s.max	
F3F2F1	$\varnothing d$	Pos.	Val.	Pos.	Val.	Pos.	Val.	Pos.	Val.	Pos.	Val.
a=0.6	0	0	0.0243	3.7325	0.0821	7.0339	0	9.0662	0.0167	11.6191	0
b=0.9	$\pi/2$	0	0.062	1.2754	0.0608	3.547	0.0768	7.0098	0	9.064	0.0161
$\beta=0.5$	π	0	0.1232	6.9491	2.00E-04	9.0479	0.0153	11.6295	5.00E-04		
fig-194	$3\pi/2$	0	0.1956	6.8459	3.00E-04	9.0227	0.0141	11.6374	9.00E-04		
	2π	0	0.2677	6.6879	2.00E-04	8.9954	0.0124	11.6513	0.0014		

Maxima minima position and values of the point spread function of three zone aperture for different degrees of variable $\varnothing d$ values for different degrees of Co-sinusoidal filter -first zone, Gaussian filter -second zone and Hanning filter in the outer zone apodization varying values of β is 0.5 and 1

1.4 Conclusions:

The present work aimed to improve the resolution of an optical system with three zone aperture by using Hanning amplitude filters, Gaussian amplitude filters, Co-sinusoidal amplitude filters and phase filters with possible combinations. It can be concluded that complex pupil filters to achieve apodization could result in an extra boost for resolution. Their superior side lobe suppression and central lobe narrowing abilities have been studied for $\beta = 0.5$ and $\beta = 1$ and $\theta_d = (-\pi/4)$ and $(\pi/3)$ values and the position of first minimum is higher than the Airy case. But these values yield a better result by obtaining steep principal maximum and highly suppressed side lobes for the whole range of interest. Overall, it should be underlined that the usage of complex pupil functions with amplitude and phase filters could result in an extra enhancement for resolution

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