RESULTS OF HEAT TRANSFER OF FREE CONVECTION FLOW OF CASSON FLUID BY USING MAGNETOHYDRODYNAMIC

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ABSTRACT

This paper presents very important analytical and numerical results for the Magnetohydrodynamic Casson fluid flow version including Hall effect and Rotation. Analytical solutions were found depending on physical parameters such as Hall parameters, Hartmann numbers, Casson parameters and rotational parameters. The effect of these on the velocity profile is shown graphically and results are discussed. The numerical results shows the velocity component decreases with the increase of the Casson fluid parameter and the rotation. Increasing the Hartmann number resists the primary velocity and increase the secondary velocity, which is the opposite for increasing inclined angle values.

KEYWORD: Magneto hydrodynamic flow, Uniform plate, Hall effect, Inclined magnetic field.

1. INTRODUCTION

Rotating fluid MHDs have attracted attention in recent decades due to their new capabilities in explaining many natural phenomena in the fields of geophysics and astrophysics. In many small-scale motions, rotation plays an important role, such as those found in centrifugal pumps and hydraulic turbines.

The unsteady flow in a rotating medium has been extensively studied since the research of Hide and Roberts [1], where that investigated the effects of coriolis force on the Stokes-Rayleigh problem on an isolated plate in the presence of a magnetic field. Chaturvedi [2] studied the effect of exponentially varying magnetic fields and gravitational forces on the flow of an incompressible, viscous, conducting fluid through an impulsively initiated infinite plate.

Puri and Kuishrestha [3] studied unsteady Hydromagnetic boundary layers in rotating media. Tokis and Geroyannis [4] further analyzed flow fluctuation on three unsteady flows of viscous, conducting and rotating fluids near horizontal planes in the presence of a transverse magnetic field. The Hall effect on MHD flow through accelerated plates was investigated by Deka [5]. Guchhait et al [6] investigated the combined effects of Hall flow and rotation on MHD flow in a rotating vertical channel. The effects of Hall current and rotation on MHD free convection through a vertical plate under a varying transverse magnetic field were studied by Hannington situma et. al. [7]

In practical situations, the magnetic field is not always horizontal or perpendicular to the axis of rotation and plane. In this chapter, it has been to investigate the effects of MHD with Hall effect and rotation in the presence of inclined magnetic fields in the flow of Casson fluids.

2. FORMULATION OF THE PROBLEM

Consider the flow of an incompressible conducting Casson fluid through an infinite plane occupying the y=0 plane. At first, the fluid and the plate rotate with a uniform angular velocity $\bar{\Omega}$ around the y - axis perpendicular to the plate. The x-axis is taken in the direction of the plane's movement and the z – axis is perpendicular to both x and y axis on the plane. For a rotating fluid, the plate starts impulsively at rest and begins to move in its plane along the x-axis with uniform acceleration. A uniform magnetic field H_0 is applied parallel to the y-axis, which makes the plate electrically non-conductive.

These Physical configuration and flow properties show the following form of velocity vector $\bar{q}$, magnetic induction vector $\bar{H}$, the uniform angular velocity $\bar{\Omega}$, electro static field $\bar{E}$ and Pressure P, thus $\bar{q}$= (u,0,w), $\bar{H}$= (H_0sinθ,H_0cosθ,0), $\bar{\Omega}$ = (0,Ω_y,0), $\bar{E}$= (E_x,0,E_z) and P is Constant. The rheological equation of state for an isotropic and incompressible flow of Casson fluid is as follow [8]

$$\tau_{ij}= \begin{cases} 2(\mu_\beta + \frac{P_y}{\sqrt{2\pi}})e_{ij}, & \pi > \pi_c \\ 2\left(\mu_\beta + \frac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}, & \pi < \pi_c \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} are the $(i,j)th$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_β is plastic dynamic viscosity of the non-Newtonian fluid and P_y is the yield stress of the fluid. The basic equations with reference to the rotating frame governing the unsteady flow in the presence of magnetic field and Hall current are as follows:

Equation of continuity

$$\nabla.\bar{q}=0 \tag{1}$$

Momentum equation

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q} + 2\bar{\Omega}\times\bar{q} = -\frac{1}{\rho}\nabla P + v\,(1+\frac{1}{\gamma})\,\nabla^2\bar{q} + \frac{1}{\gamma}(\bar{J}\times\bar{B}) \tag{2}$$

The generalized Ohm's law, neglecting ion-slip effect but taking Hall current into account is,

$$\frac{\bar{J}}{\sigma} = (\bar{E}+ \bar{q}\times \bar{B}) - \frac{\bar{J}\times\bar{B}}{n.e} \tag{3}$$

where $\sigma = \frac{e^2\tau n}{m_e}$ (is the electrical conductivity).

Here $\bar{J}$ is the current density, ρ is density, v is kinematic viscosity, e is electric charge, m_e is mass of an electron, τ is the mean collision time and n is the electron number density. As the plate is infinite, all variables in the problem are functions of y and t only. In equation (2) the values of ∇P and $(\bar{q}.\nabla)\bar{q}$ reduces to zero, since P is constant and all the variables in the problem are functions of y and t.

The initial and boundary conditions are

$u = 0$, $w = 0$ for all $t \le 0$ and for all y

$u = U_0$, $w = 0$ for all $t > 0$ and $y = 0$,

$u \to 0$, $w \to 0$ for all $t > 0$ and $y \to \infty$ (4)

Physical quantities are cast in non-dimensional form by using the following non-dimensional scheme.

$$y^* = \frac{U_0\, y}{v}, \quad u^* = \frac{u}{U_0}, \quad w^* = \frac{w}{U_0}, \quad t^* = \frac{{U_0}^2 t}{v} \tag{5}$$

Now introducing the above non-dimensional quantities in equation (2), the components are

$$\frac{\partial u}{\partial t} = (1+\frac{1}{\gamma})\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\, {H_0}^2 v}{\rho {U_0}^2(1+\omega^2\tau^2)}(u + \omega\tau w) - \frac{2v}{{U_0}^2} w\Omega_y \tag{6}$$

$$\frac{\partial w}{\partial t} = (1+\frac{1}{\gamma})\frac{\partial^2 w}{\partial y^2} - \frac{\sigma\, {H_0}^2 v}{\rho {U_0}^2(1+\omega^2\tau^2)}(\omega\tau u - w) + \frac{2v}{{U_0}^2} u\Omega_y \tag{7}$$

where the non-dimensional parameters are defined below

$M^2 = \frac{\sigma {H_0}^2 v}{\rho {U_0}^2}$ is the square of the Hartmann number,

$m = \omega\tau$ is the Hall Parameter and

$K^2 = \frac{v\Omega_y}{{U_0}^2}$ is the Rotation parameter i.e., the reciprocal of Ekmann number.

The corresponding initial and boundary conditions (4) in non-dimensional forms are

$t \le 0 : u\,(y, t) = 0;\ w\,(y, t) = 0$ for all y.

$t > 0 : u\,(0, t) = 1,\ w\,(0, t) = 0$

$$t > 0 : u\,(y, t) \to 0,\ w\,(y, t) \to 0 \text{ as } y \to \infty \tag{8}$$

3. SOLUTION OF THE PROBLEM

By introducing $q = u + iw$, equation (6) and (7) becomes

$$\frac{\partial q}{\partial t} = c\frac{\partial^2 q}{\partial y^2} - [(\frac{M^2 sin^2\theta}{1+m^2 sin^2\theta})(1 - im\sin\theta) - 2iK^2]\, q \tag{9}$$

where $c = (1+\frac{1}{\gamma})$

The initial and boundary conditions take the form

$$q(y,0) = 0,\ q\,(0, t) = 1,\ q\,(y, t) \to 0 \text{ as } y \to \infty \tag{10}$$

Using the abbreviation $\alpha = [(\frac{M^2\, sin^2\,\theta}{1+m^2 sin^2\theta})\,(1 - im\,\sin\theta) - 2iK^2]$,

Equation (2.9) can be written as

$$\frac{\partial q}{\partial t} = c\,\frac{\partial^2 q}{\partial y^2} - \alpha q \tag{11}$$

Also substitute $q\,(y, t) = e^{i\xi t}\, g(y)$ in (11), we have

$$c\, g''(y) - (i\xi + \alpha)\, g(y) = 0 \tag{12}$$

Equation (12) can be solved under the boundary conditions,

$$g\ (0) = e^{-i\xi t}, \quad g\ (\infty) = 0 \tag{13}$$

The solution is

$$g(y) = e^{-i\xi t}\ e^{\frac{-y}{\sqrt{c}}\sqrt{i\xi}+\alpha} \tag{14}$$

Hence $q\ (y, t) = e^{i\xi t}\ [e^{-i\xi t}\ e^{-y\sqrt{i\xi}+\alpha}]$ (15)

Real and imaginary parts of equation (15) are

$$u\ (y, t) = e^{-ys_1}\cos yS_2 \tag{16}$$

$$w\ (y, t) = -e^{-ys_1}\sin yS_2 \tag{17}$$

Where $a = \frac{M^2 sin^2\theta}{1+m^2 sin^2\theta}$; $b = -\ \frac{M^2 m sin^3\theta}{1+m^2 sin^2\theta} - 2K^2$;

$S_1 = \frac{1}{\sqrt{c}}\ \sqrt{\frac{a+\sqrt{a^2}+(\xi+b)^2}{2}}$; $S_2 = \frac{1}{\sqrt{c}}\ \sqrt{\frac{-a+\sqrt{a^2}+(\xi+b)^2}{2}}$

4. RESULTS AND DISCUSSION

For a clear understanding of the problem, the figure shows the effects of Hall parameter m, Hartmann number M^2, inclination angle θ, rotation parameter K^2 and Casson fluid parameter on the velocity components u and w.

Figure 1 and 2 show the primary and secondary velocities for different values of the Hartmann number. Applying a transverse magnetic field to a conductive fluid that produces a resistive force called the Lorentz force. This force tends to slow down the fluid movement in the boundary layer.

As it is clear from the above figure, as the strength of the magnetic effect increases, the primary and secondary speed increases. The effect of Hall parameters on the velocity profile is shown in figures 3 and 4. As the Hall current increases, the first and second order velocity profiles decrease. This is because as the Hall parameter increases, the effective conductivity decreases and the magnetic damping force decreases.

The velocity profiles for different values of the Casson fluid parameter and the Rotation parameter are shown in figures 5,6 and 7,8 respectively. It can be seen the by increasing the values of the Casson fluid parameter and Rotation parameter. The increase in primary velocity and decrease in secondary velocity for increasing values of angle of inclination are clearly shown in figures 9 and 10.

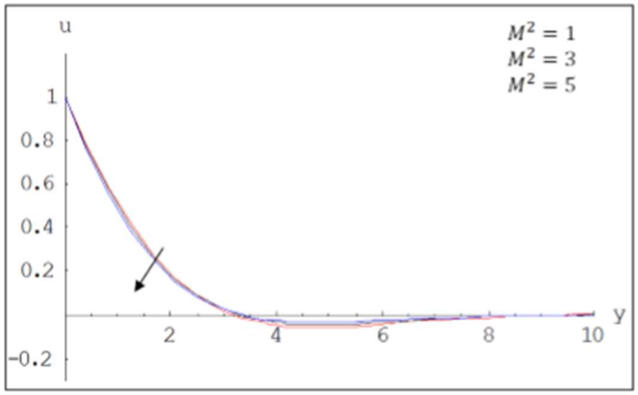

Figure 1 Effect of Hartmann number (M^2) on primary velocity profile
when $\gamma = 0.2$; $m = 1$; $K^2 = 2$; $\theta = 60^\circ$; $\xi = 1$; $K_p = 1$

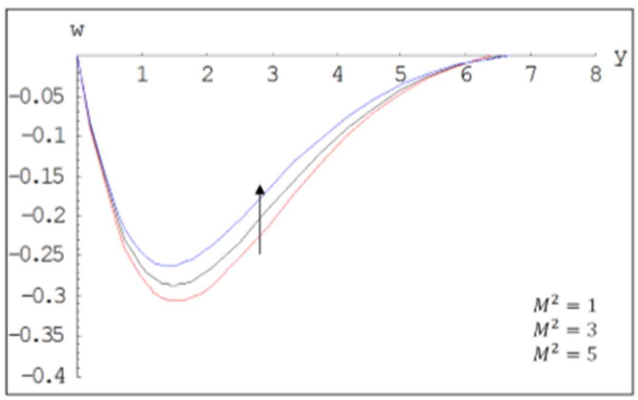

Figure 2 Effect of Hartmann number (M^2) on secondary velocity profile
when $\gamma = 0.2$; $m = 1$; $K^2 = 2$; $\theta = 60^\circ$; $\xi = 1$; $K_p = 1$

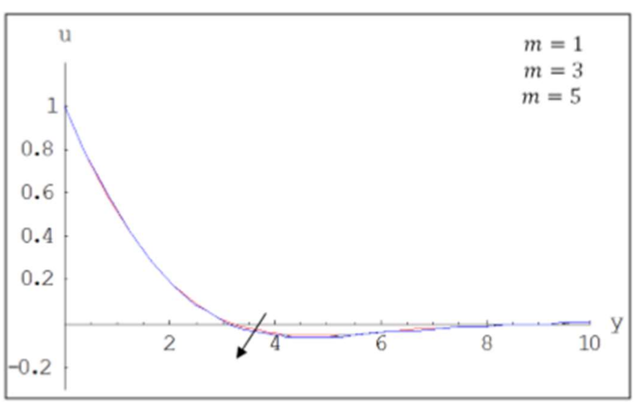

Figure 3 Effect of Hall parameter (m) on primary velocity profile

when $\gamma = 0.2$; $M^2 = 1$; $K^2 = 2$; $\theta = 60^\circ$; $\xi = 1$; $K_p = 1$

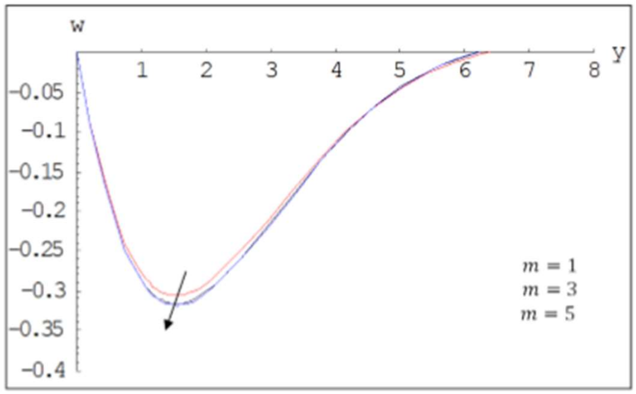

Figure 4 Effect of Hall parameter (m) on secondary velocity profile

when $\gamma = 0.2$; $M^2 = 1$; $K^2 = 2$; $\theta = 60^\circ$; $\xi = 1$; $K_p = 1$

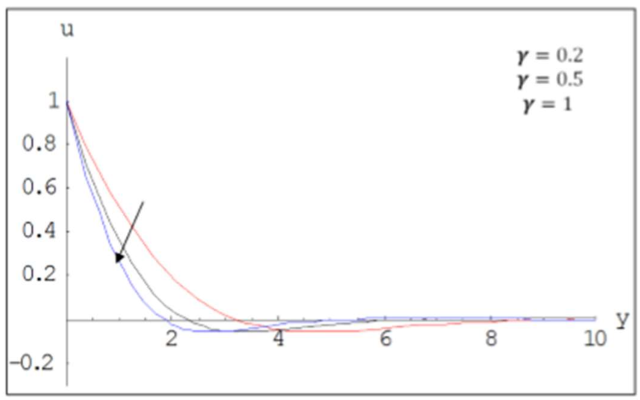

Figure 5 Effect of Casson fluid parameter (γ) on primary velocity profile

when $m = 1$; $M^2 = 1$; $K^2 = 2$; $\theta = 60^\circ$; $\xi = 1$; $K_p = 1$

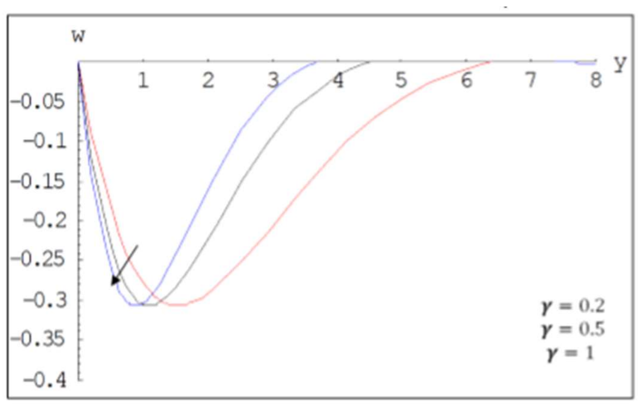

Figure 6 Effect of Casson fluid parameter (γ) on secondary velocity profile

when m = 1; M^2 = 1; K^2 = 2; θ = 60˚; ξ= 1; K_p = 1

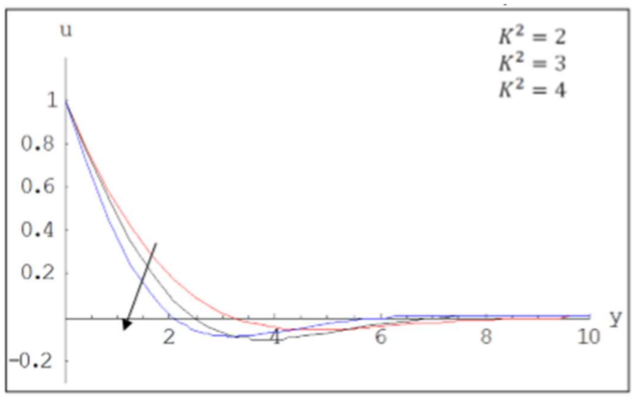

Figure 7 Effect of Rotation parameter (K^2) on primary velocity profile

when γ = 0.2; M^2 = 1; m = 1; θ = 60˚; ξ = 1; K_p = 1

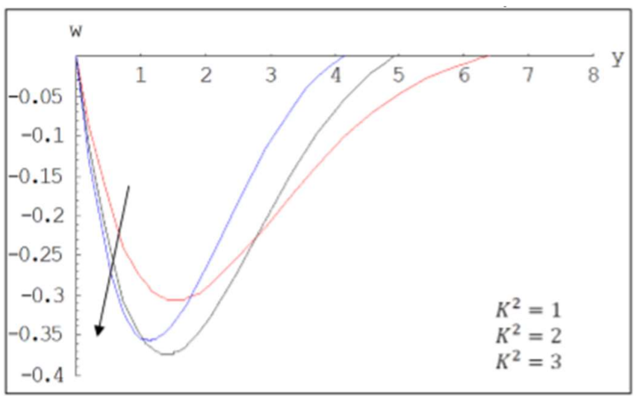

Figure 8 Effect of Rotation parameter (K^2) on secondary velocity profile

when γ = 0.2; M^2 = 1; m = 1; θ = 60˚; ξ = 1; K_p = 1

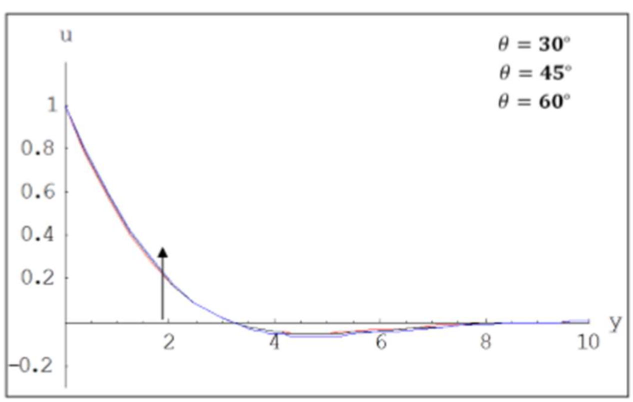

Figure 9 Effect of angle of inclination (θ) on primary velocity profile

when $\gamma = 0.2$; $M^2 = 1$; $m = 1$; $K^2 = 2$; $\xi = 1$; $K_p = 1$

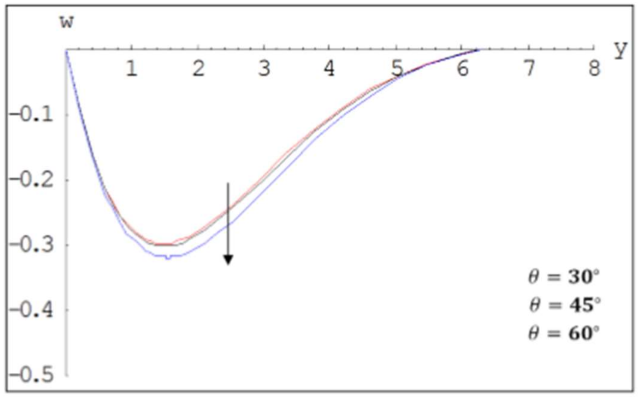

Figure 10 Effect of angle of inclination (θ) on secondary velocity profile

when $\gamma = 0.2$; $M^2 = 1$; $m = 1$; $K^2 = 2$; $\xi = 1$; $K_p = 1$

5. FUTURE RESEARCH

In future, when we enforce the free convection flow of Casson fluid, there will be an improvement of corrosion science will make a rapid growth in the commercialization. It will be a saving billions of dollars much better that the utilization. The development of Casson fluid to applying practical efficiency will be most significant in upcoming decade. The effect of MHD free convection of heat transfer of Casson fluid will be the most successful one is an upcoming decade. The technologies utilizing strange rheological behaviours of Casson fluid are still in progress and are expected to provide surprising products and applications we can benefit from in our everyday life.

6. CONCLUSION

From the above results and discussions, it can be concluded that with the increase of Hall parameter, Casson fluid parameter and Rotation parameter, the first and second order velocity components slow down. Increasing the Hartmann number resists the primary velocity and accelerates the secondary velocity but this is converse for increasing angle of inclination values.

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