FERMATEAN FUZZY SOFT COVERED CONGRUENCE RELATIONS ACTING ON A SEMI-GROUP

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Abstract: In this paper, We introduce the concept of a Fermatean fuzzy soft-covered generalized biideal on a semi-group, which is an extension of the concept of a Fermatean fuzzy soft-covered generalised biideal on a semi-group, which is an extension of Fermatean fuzzy soft biideal and characterize regular semi-groups in terms of Fermatean fuzzy soft generalized biideals. By combining fuzzy relations, composition relations and compatible relations with Fermatean fuzzy sets are framed. The notion of Fermatean fuzzy soft equivalence relation(FFSER) and Fermatean fuzzy soft compatible relation(FFSCR) on a semi-group is introduced, which is an extended notion of Fermatean fuzzy soft subgroup(FFSSG). We also provide a Fermatean fuzzy soft inverse relation, Fermatean fuzzy soft congruence on a Semi-group and its application is defined based on FFSER'S and FFCR's.

Keywords: Semigroup, soft sets, fuzzy sets, Fermatean fuzzy sets, soft semigroup, soft covered set biideal, left ideal, regular semi-group, hypothesis.

1.Introduction: The theory of collections is a necessary mathematical tool.It gives mathematical models for a class of problems that explains with exactness, precision, and uncertainty. Characteristically, non-crisp set theory is extensional. More often than not, real life problems inherently involve uncertainty, imprecision, and not clear. In particular, such classes of problems arise in economics, engineering, environmental sciences, medical sciences, and social sciences, etc. Zadeh [16] defined fuzzy set theory in his pioneering paper in 1965. In order to solve various types of uncertainties and complex MAGDM problems, the theory of fuzzy sets is proposed by Zadeh [16]. Later on, Atanassov [1] introduced the intuitionistic fuzzy set (IFS) theory to extend the concept of fuzzy sets. Yager [13] explored a typical division of these collections known as q-rung orthopair uncertainty collection in which the aggregate of the qth power of the help for and the qth power of the help against is limited by one. He explained that as 'q' builds the space of truth able orthopairs increments and therefore gives the user more opportunity in communicating their convictions about the value of membership. At the point when $q = 3$, Senapathi and Yager [10] have evoked q-rung ortho pair uncertainty collections as Fermatean uncertaintysets (FUSs).Pythagoreanuncertainty collections havestudied the concentration of many researchers within a short period of time. For example, Yager [13] has derived a helpful decision technique in view of Pythagorean uncertainty aggregation operators to deal with Pythagorean uncertainty MCDM issues. Yager and Abbasov [14] studied the pythagorean membership grades (PMGs) and the considerations related to Pythagorean uncertainty collection and presented the association between the PMGs and imaginary numbers. Senapathi and Yager [10] specified the basic activities over the FUSs and concentrated new score mapping and accuracymappings of FUSs.They proposed a technique of order preference by similarity to ideal Solution (TOPSIS) way to deal with taking care of the issue with Fermatean uncertaintydata. In an attempt to model uncertainty, the notion of fuzzy sets was proposed by Zadeh [16] as a method for representing imprecision in real-life situations.Novel cubic Fermatean fuzzy soft ideal structures were studied by[6]. This motivated D. Moldtsov's work [7] in 1999 titled soft set theory first results. Therein, the basic notions of the theory of soft sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future regarding to the theory. In this paper, we discuss the concept of a Fermatean fuzzy soft-covered generalized biideal on a semi-group, which is an extension of the concept of a Fermatean fuzzy soft biideal and characterize regular semi-groups in terms of Fermatean fuzzy soft generalized biideals. The notion of Fermatean fuzzy equivalence relation(FFER) and Fermatean fuzzy compatible relation(FFCR) on a semi-group is introduced, which is an extended notion of Fermatean fuzzy subgroup(FFSG). We also a provide Fermatean fuzzy inverse relation, Fermatean fuzzy congruence on a semi-group and its application is defined based on FFER'S and FFCR's.

2.Preliminaries:

Definition 2.1: Let S be a semigroup. By a subsemigroup of S we mean a non-empty subset of Aof such that A^2 is subset of A and by a left (resp., right) ideal ofS we mean a non-empty subset A of S such that SA is subset of A (resp., AS is subset of A).
By two-sided ideal or simply ideal we mean a subset A of S which is both a left and right ideal of S. We will denote the set of all left ideals (resp., right ideals) of S as LI (S) (resp., RI(S)).

Definition 2.2:[Zadeh, 1965]:By an uncertainty subset of a nonempty set X we mean a mapping from X to the unit interval [0,1].

Definition -2.3: [Moldtsov 1999]:LetU be an initial universe, $P(U)$be the power set of U, Ebe the set of all parameters and A ⊆ E. A soft set (f_A, E) onthe universe U is defined by the set of order pairs(f_A, E = {(e, f_A (e)): e∈ E, $f_A \in P(U)$} where f_A : E → P (U) such that $f_A(e) = \emptyset$ if e ∉ A .Here f_A is called an approximate function of the soft set.

Example-2.4: Let U = {u_1,u_2,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set ofparameters. If A = {e_1 ,e_2} ⊆ E. Let $f_A(e_1)$= {u_1, u_2 ,u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3} then we write thesoft set (f_A , E)= {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{ u_1,u_2,u_3})} over Uwhich describe the "colour of the shirts"which Mr. X is going to buy. We may represent the soft set in the following

U	e_1	e_2	e_3
u_1	1	1	0
u_2	1	1	0
u_3	1	1	0
u_4	1	0	0

Definition 2.5:[Moldtsov 1999]Let $f_A, f_B \in S(U)$. In the event

that if $f_A(x) \subseteq f_B(x)$, for all
$x \in E$, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equivalent, indicated by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

Definition -2.6:[Maji et.al 2001]Let U be an initial universe, E be the set of all parameters, and A⊆ E. A pair (F, A) is called a fuzzy soft set over U whereF: A → $\widetilde{P}$(U) is a mapping from A into $\widetilde{P}$(U), where $\widetilde{P}$(U) denotes the collection of all fuzzy subsets of U.

Example- 2.7: Consider example 2.4, here we cannot express with only two real numbers 0 and 1, we can characterize it by a membership function instead of the crisp numbers 0 and 1, which associate with each element a real number in the interval [0,1].Then(f_A, E) = {$f_A(e_1)$ = {(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)},f_A (e_2) = {(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)} is the fuzzy soft setrepresenting the "colour of the shirts" which Mr. X is goingto buy.We may represent the fuzzy soft set in the following table.

U	e_1	e_2	e_3
u_1	0.7	0.5	0
u_2	0.5	0.1	0
u_3	0.4	0.5	0
u_4	0.2	0	0

Definition 2.8: [Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean fuzzy set "F" in X is an object having the form $F = \{\langle x, m_F(x),\ n_F(x)\rangle / x \in X\}$,
where $m_F(x): X \to [0,1]$ and$n_F(x): X \to [0,1]$,including the condition
$0 \le (m_F(x))^3 + (n_F(x))^3 \le 1$, for all x ∈ X. The numbers $m_F(x)$ signifies the level (degree) ofmembership and $n_F(x)$indicate the non-membership of the element 'x' in the set F.All through this paper, we will indicate a Fermatean fuzzy set is FFS.

For any FFS 'F' and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - (m_F(x))^2 - (n_F(x))^2}$ is to find out as the degree of indeterminacy of 'x' to F. For convenience, senapathi and yager called $(m_F(x),\ n_F(x))$ a Fermatean fuzzynumber (FFN) denoted by $F = (m_F,\ n_F)$.

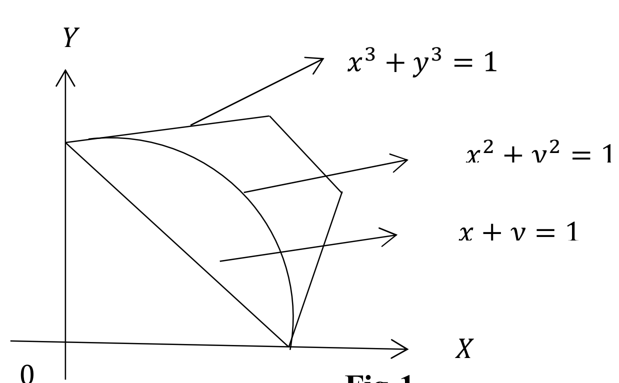
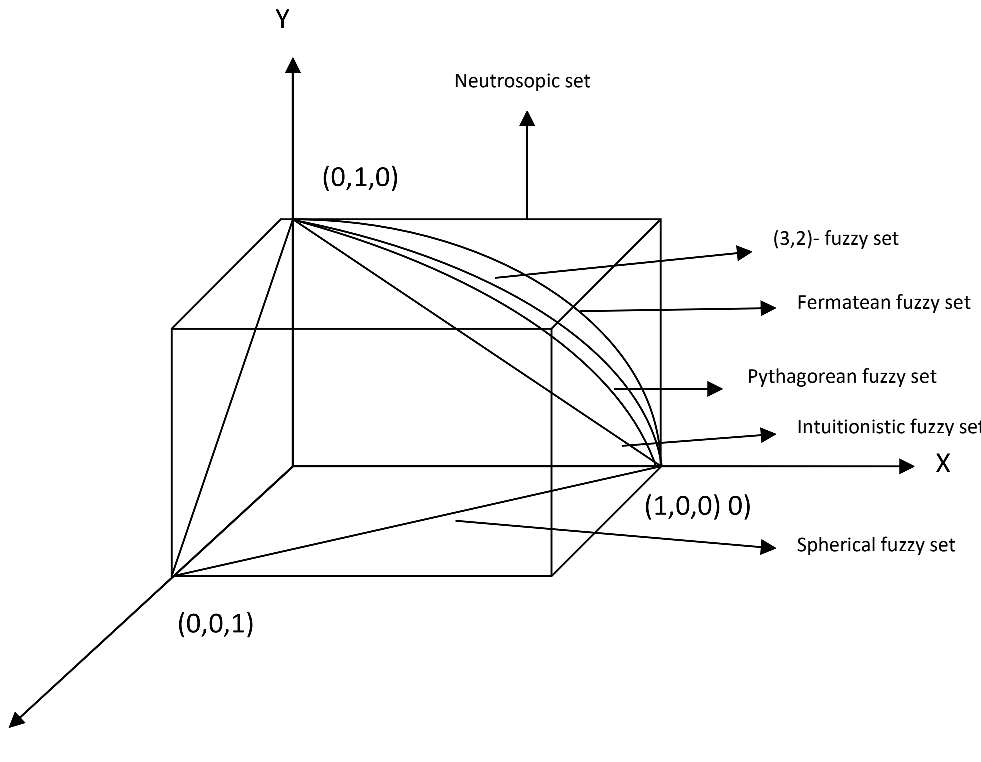


Fig-1

We will explain the membership grades (MG's) related Fermatean uncertainty collection as Fermatean membership grade.

Theorem2.9:[Senapati and Yager, 2019a] The collection of FMG's are higher than the set of pythagorean membership grades (PMG's) and bi –fuzzy membership grades (BMG's).
Proof: This improvement can be evidently approved in the following figure.



Here we find that BMG's are all points beneath the line $x + y \leq 1$, the PMG's are all points with $x^2 + y^2 \leq 1$. We see that the BMG's enable the presentation of a bigger body of non-standard membership grades than BMG's and PMG's. Based on Fermatean fuzzy membership grades, We study interval-valued Fermatean fuzzy soft sets in matrix aspect.

Definition 2.10: Let D be a non-empty crisp set and $\gamma_1 = ({\alpha_{D_1}}^3(m), {\beta_{D_1}}^3(m))$ and $\gamma_2 = ({\alpha_{D_2}}^3(m), {\beta_{D_2}}^3(m))$ be FFSs on D. Then

(i) $\gamma_1 = \gamma_2$ if and only if ${\alpha_{D_1}}^3(m) \leq {\alpha_{D_2}}^3(m)$and ${\beta_{D_1}}^3(m)) \geq {\beta_{D_2}}^3(m)$.
(ii) $\gamma_1 = \gamma_2$if and only if $\gamma_1 \subset \gamma_2$ and $\gamma_2 \subset \gamma_1$.
(iii) ${\gamma_1}^c = ({\beta_{D_1}}^3(m), {\alpha_{D_1}}^3(m))$.
(iv) $\gamma_1 \cap \gamma_2 = (\min \{ {\alpha_{D_1}}^3(m), {\alpha_{D_2}}^3(m)\}, \max \{{\beta_{D_1}}^3(m), {\beta_{D_2}}^3(m)\})$
(v) $\gamma_1 \cup \gamma_2 = (\max \{ {\alpha_{D_1}}^3(m), {\alpha_{D_2}}^3(m)\}, \min \{{\beta_{D_1}}^3(m), {\beta_{D_2}}^3(m)\})$
(vi) $[\ ]\gamma_1 = ({\alpha_{D_1}}^3(m), 1 - {\alpha_{D_2}}^3(m)\}$, $< > \gamma_1 = (1 - {\beta_{D_2}}^3(m), {\beta_{D_1}}^3(m))$.

Definition 2.11: Let $\{\gamma_i\}$ be an arbitrary family of FFS's in C, where $\gamma_i = (({\alpha_{D_i}}^3(m), {\beta_{D_i}}^3(m))$ for each $i \in J$. Then

(i) $\cap\gamma_i = (\wedge {\alpha_{D_i}}^3(m), \wedge {\beta_{D_i}}^3(m))$

(ii) $\cup\gamma_i =$ ($\vee\alpha_{D_i}{}^3$(m), $\vee\beta_{D_i}{}^3$(m)).

Definition 2.12: Let S be a semi-group and let D∈FFS (s).Then D is called a

(i) Fermatean fuzzy soft semi-group (in short FFSSG) of S if $\alpha_D{}^3(m*n) \geq \min\{\alpha_D{}^3(m), \alpha_D{}^3(n)\}$ and $\beta_D{}^3(m*n) \leq \max\{\beta_D{}^3(m), \beta_D{}^3(n)\}$, for any m,n ∈ S.

(ii) Fermatean fuzzy soft left ideal (in short FFSLI) of S if $\alpha_D{}^3(m*n) \geq \alpha_D{}^3(n)$ and $\beta_D{}^3(m*n) \leq \beta_D{}^3(n)$, for any m,n ∈ S.

(iii) Fermatean fuzzy soft right ideal (in short FFSRI) of S if $\alpha_D{}^3(m*n) \geq \alpha_D{}^3(m)$ and $\beta_D{}^3(m*n) \leq \beta_D{}^3(m)$, for any m,n ∈ S.

(iv) Fermatean fuzzy soft (two-sided) ideal (in short, FFSI) of S if it is both a Fermatean fuzzy soft left and Fermatean fuzzy soft right ideal of S.

(v)

3.Fermatean fuzzy soft covered generalized biideals

A subset group A of a semi-group S is called a bi ideal of S if ASA ⊆ A.we will denote the set of all biideals of S as BI(S).

From now on, if $\Omega_S : S \to P(U)$ is a soft set satisfying $\Omega_S(x) = \emptyset$ for all x ∈ S, then Ω_S is denoted by Θ and if $\Omega_S : S \to P(U)$ is a soft set satisfying $\Omega_S(x) = U$ for all x ∈ S , then Ω_S is denoted by S.

Lemma 3.1: Let Ω_S be any soft set over U. Then we have the following

(i) $\Theta * \Theta \dot{\supseteq} \Theta$

(ii) $(S - \Omega_S) * \Theta \dot{\supseteq} \Theta$ and $\Theta * (S - \Omega_S) \dot{\supseteq} \Theta$

(iii) $(S-\Omega_S) \cup \Theta = (S-\Omega S)$ and $(S-\Omega_S) \cap \Theta = \Theta$.

Definition 3.2: Let S be semi-group and D∈FFS(s). Then D is called a soft covered Fermatean fuzzy biideal (in short, Soft C- FFBI) of S if

(i) $\alpha_D{}^3(m*n*p) \geq \min\{\alpha_D{}^3(m), \alpha_D{}^3(p)\}$ and

(ii) $\beta_D{}^3(m*n*p) \leq \max\{\beta_D{}^3(m), \beta_D{}^3(p)\}$, for any m,n, p ∈ S.we will denote the set of all soft C-FFBIs of S as soft C- FFBI(S).

Result 3.3: Let D be a non-empty subset of a semi-group S.Then D∈ BI(S) if and only if $(\alpha_D, \beta_{D^c}) \in$ Soft C-FFBI(S).

Remark 3.4: Let S be a semi-group.

(i) If α_D is a soft C-FFLI (resp., right ideal and biideal) of S, then D = $(\alpha_D, \beta_{D^c}) \in$ Soft C-FFBI(S).(resp., FFRI(S) , FFI(S) and FFBI(S)).

(ii) If D ∈ soft C- FFBI(S) , then $\alpha_D$and β_{D^c} are soft C- FFBI(S).

(iii) If D ∈ soft C- FFBI(S), then $[\ ]_D$, $<\ >_D$ ∈ soft C- FFBI(S).

A non-empty subset A of a semi-group S is called generalized biideal if ASA ⊆A.we will denote the set of all generalized biideal of S as GBI(s).

Definition 3.5: Let S be a semi-group and let D be a soft C- Fermatean fuzzy set of S. Then D is a called soft C- Fermatean fuzzy generalized biideal (in short, soft C-FFGBI) of S if and only if m, n, p ∈ S,

(i) $\alpha_D{}^3$(m ∗n ∗p) $\geq$ min { $\alpha_D{}^3$(m), $\alpha_D{}^3$(p)} and

(ii) $\beta_D{}^3$(m ∗n ∗p) $\leq$ max { $\beta_D{}^3$(m), $\beta_D{}^3$(p)}, for any m,n, p ∈ S.we will denote the set of all soft C-FFGBIs of S as softly C- FFGBI(S).It is clear that soft C-FFBI(S) ⊆ soft C- FFGBI(S).

But the converse inclusion does not hold in general.

Example 3.6: Let S = { x, y , z ,w } be a semi-group with the multiplication table:

●	x	y	z	w
x	x	x	x	x
y	x	x	x	x
z	x	x	y	x
w	x	x	y	y

We define a complex mapping A : S → I × I as follows:
A(x) = (0.2, 0.4), A(y) = (0.3 ,0.4), A(z) = (0.2 ,0.3), A(w) = (0.4, 0.3). Then we can easily show that A is soft C-FFGBI(S) but A ∉ soft C- FFBI(S).

Remark 3.7: Let S be a semi-group.

(i) If α_A is a soft C-FFBI of S, then A ∈ soft C- FFGBI(S).

(ii) If A ∈ soft C- FFGBI(S) , then α_A and β_{A^c} are fuzzy generalized biideals of S.

(iii) If A ∈ soft C- FFGBI(S), then [ ]$_A$, < >$_A$∈ soft C- FFGBI(s).

The following two lemmas are easily seen.

Lemma 3.8: Let A be a non-empty subset of semi-group S.Then A ∈ GBI(S) if and only if (α_A, β_{A^c}) ∈ soft C-FFBI(S).

Lemma 3.9: Let S be a semi-group and let A ∈ FFS(S) . Then A∈ soft C-FFGBI(S) if and only if A∘1∼∘A ⊂ A.

4. SOFT C- FERMATEAN FUZZY REGULAR SEMIGROUPS

A semi-group S is said to be regular if for each a ∈ S, there exists an x ∈ S such that a = axa.

Proposition 4.1: Let S be a regular semi-group. Then soft C- FFGBI(S) ⊆ soft C-FFBI(S).
Proof: Let A ∈ soft C- FFGBI(S) and let a,b ∈ S. since S is regular there exists an x ∈ S such that b = bxb.Then
$\alpha_A{}^3$(a ∗ b ∗ p) = $\alpha_A{}^3$ (a (bxb) ∗ p) = $\alpha_A{}^3$(a (bx)b ∗ p) $\geq$ min {$\alpha_A{}^3$(a) , $\alpha_A{}^3$(b) } and
$\beta_A{}^3$(a ∗b ∗ p) = $\beta_A{}^3$(a (bxb) ∗p) = $\beta_A{}^3$(a (bx)b ∗ p) $\leq$ max {$\beta_A{}^3$(a) , $\beta_A{}^3$(b) }.Thus
A ∈ soft C-FFGS(S). So A∈ soft C- FFGBI(S). Hence soft C- FFGBI(S) ⊆ soft C-FFBI(S).

Proposition 4.2: Let S be a semi-group. Then S is regular if A = A ∘ 1∼ ∘ A for each A ∈ soft C- FFGBI(S).
Proof: Suppose S is regular. Let A ∈ soft C- FFGBI(S) and let a ∈ S. Since S is regular, there exists an x ∈ S such that a = axa. Then

$$\alpha_{(A \circ 1\sim \circ A \circ p)}{}^3 (a) = \max_{a=yz}[\min \{ \alpha_{A \circ 1\sim}{}^3(y) , \alpha_A{}^3(z) \}]$$
$$\geq \min \{ \alpha_{A \circ 1\sim}{}^3(ax) , \alpha_A{}^3(a) \} \quad (\text{since } a = axa)$$

$= \max_{ax=ef} [\min \{ \alpha_A{}^3 (e), \alpha_{1\sim}{}^3(f), \alpha_A{}^3 (a) \}]$
$\geq \min \{ \alpha_A{}^3 (a), \alpha_{1\sim}{}^3(x), \alpha_A{}^3 (a) \}$
$= \min \{ \alpha_A{}^3 (a), \alpha_A{}^3 (a) \} = \alpha_A{}^3 (a)$ and
$\beta_{(A\circ 1\sim \circ A\circ p)}{}^3 (a) = \min_{a=yz} [\max \{ \beta_{A\circ 1\sim}{}^3(y), \beta_A{}^3(z) \}]$
$\leq \max \{ \beta_{A\circ 1\sim}{}^3(y), \beta_A{}^3(a) \}$ (since a = axa)
$= \min_{ax=ef} [\max\{\beta_A{}^3 (e), \beta_{1\sim}{}^3(f), \beta_A{}^3 (a)\}]$
$\leq \max \{ \beta_A{}^3 (a), \beta_{1\sim}{}^3(x), \beta_A{}^3 (a) \} = \max \{ \beta_A{}^3 (a), \beta_A{}^3 (a) \} = \beta_A{}^3 (a)$
by lemma-3.9, A∘1∼∘A ⊂ A. Hence A= A∘1∼∘A.

Theorem-4.3: A semi-group S is regular if and only if (soft C- FFGBI(S), ∘) is a regular semi-group.

Theorem 4.4:A semi-group S is regular if and only if for each A ∈ soft C- FFGBI(S) and B ∈ soft C- FFI(S), A∩B = A∘B∘A.
Proof: (⇒): Suppose S is regular. Let A ∈ soft C- FFGBI(S) and B ∈ soft C- FFI(S).
Then by lemma 3.9 ,A∘ B ∘ A ⊂ A∘1∼ ∘ A ⊂ A. Also we know that 1∼ ∘ A ⊂ A and A∘1∼ ⊂ A. So A ∘ B ∘ A ⊂ A ∩ B. Now let a ∈ S. since S is regular, there exists an x ∈ S such thata = axa (= axaxa). Since B ∈ soft C- FFI(S).
$\alpha_B{}^3(x\, ax * p) \geq \alpha_B{}^3 (a\,(axa * p) \geq \alpha_B{}^3 (a)\}$ and
$\beta_B{}^3(a *b * p) \leq \beta_B{}^3 (a\,(axa * p) \geq \beta_B{}^3 (a)\}$. Then
$\alpha_{(A\circ B\circ A\circ p)}{}^3 (a) = \max_{a=yz}[\min\{\alpha_A{}^3(y), \alpha_{B\circ A}{}^3(y)\}$
$\geq \min \{ \alpha_A{}^3(a), \alpha_{B\circ A}{}^3(xaxa)\}$ (since a = axaxa)
$= \min\{\alpha_A{}^3(a), \max_{xaxa=ef}[\min\{\alpha_A{}^3(e), \alpha_{B\circ A}{}^3(f)\}]\}$
$\geq \min \{ \alpha_A{}^3(a), \alpha_B{}^3(xax), \alpha_A{}^3(a) \}$
$\geq \min\{ \alpha_A{}^3(a), \alpha_B{}^3(a), \alpha_A{}^3(a) \}$
$= \min \{\alpha_A{}^3(a), \alpha_B{}^3(a)\} = \alpha_{A\cap B}{}^3(a)$ and
$\beta_{(A\circ B\circ A\circ p)}{}^3 (a) = \min_{a=yz}[\max\{ \beta_A{}^3(y), \beta_{B\circ A}{}^3(y)\}]$
$\leq \max \{ \beta_A{}^3(a), \beta_{B\circ A}{}^3(xaxa) \}$ (since a = axaxa)
$= \max\{\beta_A{}^3(a), \min_{xaxa=ef}[\max\{\beta_A{}^3(e), \beta_{B\circ A}{}^3(f)\}]\}$
$\leq \max \{ \beta_A{}^3(a), \beta_B{}^3(xax), \beta_A{}^3(a) \}$
$\leq \max \{ \beta_A{}^3(a), \beta_B{}^3(a), \beta_A{}^3(a) \}$
$= \max \{\beta_A{}^3(a), \beta_B{}^3(a)\} = \beta_{A\cap B}{}^3(a)$
So A ∩ B ⊂ A ∘ B ∘ A . Hence A ∘ B ∘ A = A ∩ B.
(⇐) : Suppose the necessary condition holds.It is clear that 1∼ ∈ soft C- FFGBI(S). Let A ∈ soft C − FFGBI(S). soft C- FFGBI(S). Then, by the hypothesis, A = A ∩ 1∼ = A∘1∼ ∘ A. Hence by theorem 4.3, S is regular.

Result 4.5 : Let S be a semi-group. Then the following are equivalent;
(i) S is regular
(ii) A∩L ⊂ AL for each A ∈ soft C- FFGBI(S) and each L ∈ soft C- FFGBLI(S).
(iii) R∩A∩L ⊂ RAL for each A ∈ soft C- FFGBI(S) and each L ∈ soft C- FFGBLI(S) and each R ∈ soft C- FFGBRI(S).

Theorem 4.6: Let S be a semi-group. Then the following are equivalent;

(i) S is regular

(ii) A∩B ⊂ A∘B for each A ∈ soft C- FFGBI(S) and each B ∈ soft C- FFGBLI(S)

(iii) A∩B ⊂ A∘B for each A ∈ soft C- FFGBI(S) and each B ∈ soft C- FFGBRI(S)

(iv) C∩A∩B⊂C∘A∘B for each A ∈ soft C- FFGBI(S) and each B ∈ soft C- FFGBLI(S) and C ∈ soft C- FFGBRI(S).

(v) C∩A∩B ⊂ C∘A∘B for each A ∈ soft C- FFGBI(S) and each B ∈ soft C- FFGBRI(S).

5. FERMATEAN FUZZY SOFT EQUIVALENCE RELATIONS

Definition 5.1: Let C be a crisp set. Then a complex mapping R = $(\alpha_R{}^3, \beta_R{}^3)$: C × C → I × I is called Fermatean fuzzy soft relation (FFR) on C if $\alpha_R{}^3$ (m, n) + $\beta_R{}^3$ (m, n) ≤ 1 for each (m, n) ∈ $C_1 \times C_2$, (ie) R ∈ FFSS ($C_1 \times C_2$).

Definition 5.2: Let R ∈ FFSR(C) .Then the inverse of R, R^{-1} is defined by R^{-1}(m,n) = R (n ,m) for any m,n ∈ C.

Definition 5.3: Let C be a set and let R, Q ∈ FFSR (C).Then the composition of R and Q, Q∘R is defined as follows; for any m ,n ∈ C

$\alpha_{Q\circ R}{}^3$(m,n) = min [$\alpha_R{}^3$(m ,z) , $\alpha_Q{}^3$(z ,n)] as z ∈ C and

$\beta_{Q\circ R}{}^3$(m, n) = max[$\beta_R{}^3$(m ,z) , $\beta_Q{}^3$(z ,n)] as z ∈ C.

Definition 5.4: A Fermatean fuzzy soft relation R on a set C is called a Fermatean fuzzy soft covered equivalence relation (in short FFER) on C if it satisfies the following conditions;

(i) R is a Fermatean fuzzy soft covered reflexive: R(m ,n) = (1, 0) for any m ,n ∈ C.

(ii) R is a Fermatean fuzzy covered symmetry : R^{-1} = R

(iii) R is a Fermatean fuzzy covered transitive : R ∘ R ⊂ R.

We will denote the set of all FFER's on C as FFE (C).

Proposition 5.5 Let R be a Fermatean fuzzy soft covered equivalence relation on a set C. Then the followings hold;

(i) $R_m = R_n$ if and only if R(m , n) = (1, 0) , for any m , n ∈ C.

(ii) R(m , n) = (0, 1) if and only if $R_m \cap R_n = 0\sim$, for any m , n ∈ C.

(iii) ∪ R_m = 1 ∼.

(iv) there exists a surjection ϕ: C → C/R defined by ϕ (m) = R_m ,for each m ∈ C.

3.3 Definition 5.6: Let C be a set and R ∈ FFSR (C) . let $\{R^{\alpha}\}_{\alpha\in\Gamma}$ be the family of all the FFER's on C ⊂ R. Then $\cap_{\alpha\in\Gamma}$ is called FFSER generated by R and denoted by R^e.

It is easily seen that R^e is the smallest Fermatean fuzzy soft covered equivalence relation containing R.

3.4 Definition 5.7: Let C be a set and R ∈ FFSR (C). Then the Fermatean fuzzy soft covered transitivity closure of R, denoted by R^∞, is defined as follows $R^\infty = \cup R^i$, i ∈ N, where R = R ∘ R ∘ … ∘ R (i factors).

Proposition5.8 : Let C be a set and R.θ∈FFSE(C), we do fine,

max{R. θ} as follows,R⋁θ = $\{R\cup\theta\}^{\alpha}$ ie R⋁θ = $\bigcup_{n\in N}\{R\cup\theta\}^{n}$.Then R⋁θ ∈ FFE(α)

Proposition 5.9: Let P and θ be any FFSERs on a set C of R◦ θ ∈ FFsE(C), then R◦ θ = R⋁θ, when R⋁θ denotes the least upper bound for {P. θ} with respect to the conclusion.

Proposition5.10: Let C be a set of Rθ ∈ FFsE(C), Then R⋁θ = $\{R\circ\theta\}^{\alpha}$

Proposition 5.11 : Let C be a set, if Rθ ∈ FFsE(C), such that, R◦ θ = θ◦R, then R⋁θ = R◦θ.

Definition5.12: A FFsR 'R' on a groupoid S is said to be,

(i) Fermatean fuzzy soft left compatible if $\alpha_R^3(x, y) \le \alpha_R^3(zx, zy)$ and $\beta_R^3(x, y) \ge \beta_R^3(zx, zy)$ for any x, y, z ∈ S

(ii) Fermatean fuzzy soft right compatible if$\alpha_R^3(x, y) \le \alpha_R^3(zx, zy)$ and $\beta_R^3(x, y) \ge \beta_R^3(zx, zy)$ for any x, y, z ∈ S

(iii) Fermatean fuzzy soft compatible if $\alpha_R^3(x,y) \wedge \alpha_R^3(z, t) \le \alpha_R^3(zx, yt)$ and $\beta_R^3(x,y) \vee \beta_R^3(z, t) \ge \beta_R^3(zx, yt)$ for any x, y, z ∈ S.

Definition 5.13: A FFSER on a groupoid 'S' is called on,

(i) Fermatean fuzzy soft left congruency (in short FFLC) if it is Fermatean fuzzy soft left compatible.

(ii) Fermatean fuzzy soft right congruency (in short IFRC) if it is Fermatean fuzzy soft right compatible

(iii) Fermatean fuzzy soft congruency (in short FFC) if it is Fermatean fuzzy soft compatible.

Proposition5.14: Let R and θ be Fermatean fuzzy soft compatible related as a groupoid 'S'. Then θoR is also Fermatean fuzzy soft compatible related to 'S'.

3.12 Proposition 5.15 : Let R and θ be Fermatean fuzzy soft compatible with a groupoid 'S'. Then the full conditions are verified.

(i) θ◦R ∈ FFSC(S) (res FFSLCS and FFSRCS)

(ii) θ◦R ∈ FFS(S)

(iii) θ◦R is a Fermatean fuzzy symmetric

(iv) θ◦R = R◦θ

3.13 Proposition 5.16 : Let 'S' be a semi-group and let θ, R ∈ FFSC(S) of R◦θ = θ◦R, then R◦θ ∈ FFSC(S).

Theorem5.17: Let 'S' be a semi-group (ie., clear that Δ, ∇∈ IFSC(S), then, (IFSC(S), ∧, ∨) is a compatible lattice write Δ and∇as the co-set and treated elements if IFSC (S).∈ S.

Let R be on Fermatean fuzzy soft congruency on a semi-group 'S' and let a ∈S. Then FFS, R_a is 'S' is called Fermatean fuzzy soft congruency class if 'R' containing a ∈ S and we will denote the set of Fermatean fuzzy soft congruency classes of R as S/R.

Proposition 5.18 : Let ‘S’ be a regular semi-group and let R ∈ FFC(S). If R_a is an idempotent element of S/R, then these exists an idempotent e ∈ S such that $R_e = R_a$.

For a semi-group ‘S’, it is clear that FFSC(S) is a partially reduced set (POSET) by the inclusion relation ‘C’ Therefore for any Pθ ∈ FFSC(S), Pnθ is the greatest cover bound of ‘P’ and ‘θ’ is (FFPC(S), C) PUθ ∈ IFC(S) is general.

Lemma 5.19: Let ‘S’ be a semi-group and that P, θ, ∈ FFC(S) we do fine P⋁θ, as follows:
P⋁θ = P⋁θ ie., $P\vee\theta = \cup_{n\in N}\{PUP\vee\theta\}^n$ then P⋁θ∈FFC(S).
Proof: By proposition-1 it is clear that P⋁θ∈FFC(S). Let x, y, t∈ S, since ‘P’ and ‘θ’ are Fermatean fuzzy left compatible.

$$\alpha_{P\vee\theta}{}^3(x,y) = \min_{n\in N}[\alpha_P{}^3(x,y), \alpha_\theta{}^3(x,y)]^n$$
$$\geq \min[\alpha_P{}^3(tx,ty), \alpha_\theta{}^3(tx,ty)]^n$$
$$= \alpha_{P\vee\theta}{}^3(tx,ty) \text{ and}$$
$$\beta_{P\vee\theta}{}^3(x,y) = \max_{n\in N}\left[\beta_P{}^3(x,y), \beta_\theta{}^3(x,y)\right]^n$$
$$\leq \max\left[\beta_P{}^3(tx,ty), \beta_\theta{}^3(tx,ty)\right]^n$$
$$= \beta_{P\vee\theta}{}^3(tx,ty) \text{ and}$$

Thus P⋁θ is Fermatean fuzzy left soft compatible, similarly ut cab be easily seen that P⋁θ is Fermatean fuzzy soft exist compatible. Hence P⋁θ ∈ FFC(S).

The following is the immediate result of Lemma 5.19.

Theorem 5.20: Let ‘P’ and ‘θ’ be any Fermatean fuzzy soft compatible with semi-group. If Poθ is a Fermatean fuzzy soft congruency on ‘S’, Then Poθ = P⋁θ denotes, the least upper bound for {P, θ} with respect to ‘S’. .

The following is the immediate result of proposition-5.8 and proposition-5.11

Proposition 5.21: Let ‘S’ be a semi-group if Pθ ∈ IFSC(S), then $P \vee \theta = (P\circ \theta)^\alpha$

Theorem 5.22: Let ‘S’ be a semi-group, then (FFSC(S) ⋀, ⋁) is a complete lattice with Δ and as the ∇ and greatest elements of FFSC(S).

Proposition 5.23: Let ‘P’ and ‘θ’ be any Fermatean fuzzy soft compatible with a group ‘G’, then R◦θ = θ◦P, hence by propostion – 5.11 and theorm-5.21 P◦θ = P⋁θ.

Proof: let x, y∈ G.Then

$$\alpha_{P\circ\theta}{}^3(x,y) = \max_{x\in G}(\min\{\alpha_\theta{}^3(x,y), \alpha_P{}^3(x,y)\})$$
$$= \max_{x\in G}\left(\min\left\{\alpha_\theta{}^3(x,y), \alpha_\theta{}^3\left(z^{-1},z^{-1}\right), \alpha_\theta{}^3(x,y), \alpha_P{}^3(x,y), \alpha_P{}^3\left(z^{-1},z^{-1}\right)\right\}\right)$$
$$\leq \max_{z\in G}\left(\min\left\{\alpha_\theta{}^3\left(y\,z^{-1}x, y\right), \alpha_P{}^3\left(x, y\,z^{-1}x\right)\right\}\right)$$
$$\leq \max_{y\,z^{-1}x\in G}\left(\min\left\{\alpha_P{}^3\left(x, y\,z^{-1}x\right), \alpha_\theta{}^3\left(y\,z^{-1}x, y\right)\right\}\right)$$
$$= \alpha_{\theta\circ P}{}^3(x,y) \text{ and}$$
$$\beta_{P\circ\theta}{}^3(x,y) = \min_{x\in G}\left(\max\left\{\beta_\theta{}^3(x,y), \beta_P{}^3(x,y)\right\}\right) = \beta_{\theta o P}{}^3(x,y)$$

Thus p ∘ θ ⊂ θ ∘ p. Similarly θ ∘ p ⊂ p ∘ θ, Hence $p \circ \theta = \theta \circ p$

Conclusion: We introduce the concept of a Fermatean fuzzy soft covered generalized biideal on a semi-group, which is an extension of the concept of a Fermatean fuzzy soft biideal and characterize regular semi-groups interms of Fermatean fuzzy soft generalized biideals.

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