A Review and Genetic Algorithm-Based Adaptive Window Optimization of the Stockwell Transform

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Abstract
This article examines the evolution of the Stockwell Transform (ST) over the years and proposes a new optimization method, called the Genetic Algorithm Optimization Approach (GAOA), designed to improve the analysis of complex signals by adjusting the standard deviation of the window function according to their characteristics. For example, this is of interest in case of electric signals, e.g., when targeting to ensure electrical power quality. At first, the present paper reviews the different variants of the ST that exist in the literature, addressing the generalized versions as well as the optimized ones and their applications in different fields. After addressing their advantages and limitations, we present the proposed GAOA, which addresses an adaptation of the signals' standard deviation of the window function to optimize time and frequency resolution. The comparison with other variants of the ST highlights the potential advantages of GAOA for more accurate analysis of complex signals.
Keywords: Stockwell transform, genetic algorithm, power quality, electrical disturbances.

1. Introduction

Over the past decades, signal analysis techniques have evolved considerably to meet the growing need for accurate characterization of complex and non-stationary phenomena. The development of advanced methods for analyzing time-varying signals has become a crucial research topic in many scientific and engineering fields, such as biomedical signal processing, geophysics, communications, and mechanical systems. These signals often exhibit transient behaviors or frequency variations that cannot be effectively represented by classical analysis methods. Traditional approaches such as the Fourier Transform (FT) [1–3] decompose a signal into a sum of sinusoids of different frequencies, providing a clear frequency spectrum but no information about the temporal evolution of the signal components. However, many real-world signals are non-stationary, which limits the use of FT for extracting relevant transient information [4,5]. To overcome this limitation, the Short-Time Fourier Transform (STFT) [6–8] was introduced, providing both time and frequency localization through a sliding analysis window. Although effective for short-duration transformations, the STFT suffers from a fixed window width, which limits its ability to adapt to varying signal characteristics.
To address this issue, the Wavelet Transform (WT)[9–14] was proposed, offering a more flexible analysis by using scalable window functions. The WT enables multi-resolution analysis and provides valuable insights into both short-term and long-term components of signals. Nevertheless, the WT is sensitive to noise and sometimes lacks direct interpretability in the frequency domain [15]. An alternative and powerful tool for time-frequency analysis is

the ST [16–24]. Introduced as an extension of the WT, the ST combines the advantages of both the FT and WT. It provides a phase-corrected representation that retains precise frequency localization while preserving temporal information. Over the years, several variants of the ST have been developed to improve its performance and adaptability for different classes of signals. These include generalized versions and optimized approaches designed to enhance resolution and robustness.

Building on these developments, the present work examines the evolution of the ST and proposes a new optimization method, the Genetic Algorithm Optimization Approach (GAOA). This method is inspired by the work of Moukadem et al. [25], who introduced additional parameters into the ST window function to improve its adaptability. The GAOA extends this idea by dynamically adjusting the standard deviation of the ST window through a genetic algorithm, thereby optimizing both time and frequency resolutions according to the characteristics of the analyzed signal.

The remainder of this paper is organized as follows: Section 2 presents the standard ST, outlining its advantages and limitations. Section 3 reviews the ST various variants developed over time. Section 4 details the proposed GAOA and its specific implementation principles. Finally, Section 5 concludes this paper and opens up perspectives for future works.

2. The Stockwell Transform

The Stockwell transform (ST) was introduced by Stockwell et al. in 1996 [22]. In the field of signal processing, the ST is a multi-resolution time-frequency analysis technique. It uses a Gaussian window whose width varies inversely with frequency:

- Wide for low frequencies → better frequency resolution
- Narrow for high frequencies → better time resolution.

By adjusting the width of the window inversely proportional to the frequency, S-transform processing offers increased temporal resolution for high frequencies, as well as high frequency resolution for low frequencies [22,26,27].

Thanks to this property, the ST can be interpreted as:

- A frequency-dependent version of the short-term Fourier transform (STFT)
- A phase-corrected wavelet transform (WT) [28–30].

These characteristics make the ST a versatile tool for signal analysis.

2.1. The continuous S-transform

The expression of the S-transform (ST) [22]is given by the relation:

$$S(\tau,f)=\int_{-\infty}^{+\infty} h(\tau)\left|f\right| e^{-\frac{(t-\tau)^2 f^2}{2}} e^{-j2\pi ft}dt \qquad (1)$$

where f is the frequency, and t is time. Indeed, the Gaussian window is defined by:

$$w(t,f)=\left|f\right| e^{\frac{-t^2 f^2}{2}} \qquad (2)$$

The window satisfies the condition:

$$\int_{-\infty}^{+\infty} g(t,f)dt=1 \qquad (3)$$

The inverse S transform, which allows the reconstruction of the original signal, is expressed as:

$$h(t)=\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} S(\tau,f)e^{j2\pi ft}d\tau\right]df \qquad (4)$$

This continuous formulation ensures that the S-transform preserves both the amplitude and phase of the analyzed signal, offering an interpretable time–frequency representation.

2.2. Frequency domain computation

To reduce computational cost, the S-transform can be efficiently computed in the frequency domain using the Fourier Transform (FT). If H(f) represents the FT of h(t), the S-transform can be expressed as:

$$S(t,f) = F^{-1}\left[H(f')e^{-\frac{2\pi^2(f-f')^2}{f^2}} \right] \tag{5}$$

where F^{-1} denotes the inverse Fourier transform. This formulation simplifies implementation and is widely used in digital signal analysis.

2.3. The discrete ST

For discrete-time signals $h[kT]$ where k = 0, 1, …, N-1, the discrete Fourier transform is written as [29,31]:

$$H\left[\frac{n}{NT}\right] = \frac{1}{N}\sum_{p=0}^{N-1} h[kT] e^{\left(-\frac{i2\pi nk}{N}\right)} \tag{6}$$

The discrete ST is then defined by:

$$S[k,n] = \sum_{m=0}^{N-1} H[m+n] e^{-\frac{2\pi^2 m^2}{n^2}} e^{j2\pi mk/N} \tag{7}$$

For n = 0, the ST reduces to the mean value of the signal. This discrete formulation preserves the essential time–frequency characteristics while making the computation feasible for sampled signals.

2.4. General limitations of the ST

Despite its advantages, the ST still presents some inherent limitations. Being derived from the Fourier framework, it remains subject to issues such as implicit periodicity, edge effects, and dependence on sampling conditions. Furthermore, the use of a fixed Gaussian window may not be optimal for signals with rapidly changing or multi-component characteristics, which can limit time–frequency precision.

To address these constraints, several variants and optimized versions of the S-transform have been proposed, aiming to improve adaptability and resolution. The next section reviews these developments.

3. The modified S-transform in the literature

Over time, the standard ST has been revisited to better adapt its performance to diverse signal types and analysis contexts. Researchers have introduced several modifications aimed at enhancing its time–frequency resolution, energy concentration, and adaptability. These developments fall mainly into two categories: generalized versions, which extend the mathematical formulation of the transform, and optimized versions, which adjust its parameters dynamically to improve analysis accuracy. This section presents these two main families of approaches and their respective roles in advancing the capabilities of the ST.

3.1. The generalized versions of ST

Stockwell's original ST uses a Gaussian window whose width varies inversely with frequency, ensuring good frequency resolution for low frequencies and good time resolution for high frequencies. However, this approach, although robust, does not always take into account the specific characteristics of the analyzed signal. To overcome this limitation, several variants of the ST, known as generalized, have been proposed, modifying the shape, width, or symmetry of the window.

3.1.1. Initial version: Mansinha et al. [32] proposed the first version of the Generalized ST (GST), which is defined by:

$$S(\tau,f)=\int_{-\infty}^{+\infty} h(t)\frac{\left|\frac{f}{k}\right|}{\sqrt{2\pi}}e^{-\frac{(\tau-t)^2 f^2}{2k^2}}e^{-i2\pi t}dt \tag{8}$$

The objective is to improve frequency resolution by increasing *k*, while adjusting it according to the analysis needs. This provides direct control of the time–frequency trade-off, although excessively large *k* values degrade the temporal localization of fast events.

3.1.2. Generalized window: Other authors [31,33,34]proposed replacing the Gaussian window with a more general window $\mathrm{w(t, f, p)}$, where p represents a set of parameters controlling the shape of the window. This approach increases the flexibility of the transform and allows customized windows suited to specific applications such as transient detection. However, parameter selection is often empirical and may degrade time–frequency concentration if not optimized, which motivated later works on automatically optimized versions of the ST.

3.1.3. Complex modulated window: Pinnegar et al. [35] proposed a modified version of the ST using a complex modulated Gaussian window. In this approach, a phase modulation term is introduced into the window function, controlled by a positive parameter σ that determines the degree of modulation. This modification makes the transform conceptually close to the continuous wavelet transform, while preserving the fixed phase reference of the original ST. The complex modulation improves time–frequency resolution, particularly for signals containing sinusoidal components at multiple scales, allowing finer discrimination of specific event signatures. However, this improvement comes at the cost of added phase complexity, which can make the visual interpretation of results more challenging compared to the conventional real-valued Gaussian window.

3.1.4. Integration of the number of periods: Mansinha et al. [36] proposed a modification of the ST that introduces the number of periods of the Fourier sine wave into the definition of the Gaussian window. This parameter represents the number of oscillations included within one standard deviation of the window. By adjusting this parameter, the window width can be directly controlled: reducing it improves time resolution, while increasing it enhances frequency resolution. However, setting it too low limits the number of cycles within the window, which can lead to poor frequency definition and a less interpretable spectrum. This approach thus offers a straightforward way to balance time and frequency resolution, though it requires careful parameter tuning to maintain meaningful time–frequency representation.

3.1.5. **Asymmetric and bi-Gaussian windows:** Several researchers have explored asymmetric window functions to improve the temporal localization of events in the ST.

a. Asymmetric window: McFadden et al. proposed an asymmetric window combining a Gaussian front and an exponential tail[37]. Although this structure extends event signatures beyond their actual duration, it improves sensitivity to certain transient components. However, the elongated tail can distort the timing of detected events.
b. Hyperbolic window: To address the issue, Pinnegar et al. [38] developed a hyperbolic window, derived from the GST. It offers a more asymmetric shape at low frequencies, enhancing temporal resolution during event initiation while maintaining acceptable frequency precision.
c. Bi-Gaussian window: Pinnegar et al. [34,39] also introduced the bi-Gaussian window, composed of two half-Gaussians with different spreads on each side of the time origin. This configuration improves event onset localization and provides finer detail than the classical Gaussian ST, though it may introduce high-frequency artifacts**.**

Overall, these asymmetric and bi-Gaussian approaches aim to enhance the detection of short or transient phenomena by refining the time localization of event signatures in the time–frequency plane.

3.1.6. Width parameter λ: Xu et al. [40] introduced a GST that incorporates a width parameter λ into the window function to control its rate of variation. This parameter allows direct adjustment of the time–frequency resolution of the transform. A smaller λ enhances frequency resolution, making it suitable for analyzing harmonic or steady components, while a larger λ improves time resolution, which is advantageous for detecting transient or complex phenomena. The choice of λ thus represents a trade-off between temporal precision and spectral accuracy, enabling the transform to be adapted to different signal analysis requirements.

3.1.7. **Multi-resolution generalized S-transform (MGST):** Nantian Huang et al. [41] proposed the Multi-Resolution GST(MGST) to enhance the analysis and classification of complex signals. In this approach, the GST matrix is divided into three frequency zones low, medium, and high each using adapted window parameters to better represent signal characteristics. The method differs from previous versions in several aspects:

- The window width factor is adjusted for each frequency band, improving discrimination of complex events.
- Feature extraction is performed separately within each zone, reducing interference and computational cost.
- A cutoff threshold Ts, optimized via a modified particle swarm optimization (PSO) algorithm, enhances noise robustness.

Simulation results show that MGST improves both accuracy and noise resistance compared to conventional ST methods. Future work aims to further optimize window parameters and reduce computation time.

3.1.8. **Summary of the generalized versions of ST:** The generalized versions of the ST were developed to overcome the rigidity of the original formulation, particularly its fixed Gaussian window, which limits adaptability to different signal types. These variants introduce additional parameters or modified window functions such as complex, asymmetric, bi-Gaussian, hyperbolic, or frequency-dependent windows to allow greater control over time–frequency resolution. The main objective of these generalized forms is to adapt the window width and shape according to signal characteristics, thereby improving the representation of transient and non-stationary phenomena. Some methods, like the multi-resolution GST, further divide the frequency spectrum into multiple bands, each with tailored window parameters, enhancing the analysis of signals with mixed or overlapping components.

Overall, the GST family significantly expands the flexibility of the original ST, offering better adaptability, resolution, and interpretability, though often at the cost of increased computational complexity or parameter tuning requirements.

3.2. The optimized versions of ST

Over the past two decades, the ST has undergone numerous adaptations designed to overcome certain limitations of its initial formulation. This work has mainly focused on improving time-frequency resolution, enhancing noise robustness, and making the tool more flexible for analyzing varied, often non-stationary signals. Researchers have explored various avenues: modifying the window function, adaptively adjusting its parameters, integrating optimization criteria derived from artificial intelligence, or combining it with other time-frequency approaches. In what follows, we present a structured overview of these contributions, highlighting the objectives pursued, the methodological choices made, and the advantages obtained.

3.2.1. IST – Improved S-transform: Sejdic et al.[42] proposed an Improved ST (IST) by modifying the standard deviation of the Gaussian window through a parameter p controlling its width. By adjusting p, the method achieves better time–frequency concentration. The optimal value of this parameter is determined using the concentration measure introduced by Stankovic et al. [43], known for its superior performance.

Two optimization strategies are possible:

- using a constant parameter for the entire signal, suitable for signals with steady frequency components;
- or a time-varying parameter, adapted for signals with changing frequency content. Building on this idea, Sanchez et al. [44] applied a genetic algorithm to automatically optimize the ST for the analysis and classification of complex disturbances, showing improved accuracy compared to the classical version.

3.2.2. MST – Modified S-transform: Building on the adaptive adjustment of the Gaussian window, George et al. [45] introduced the Modified ST (MST), which incorporates two parameters η (slope) and b (intercept) to jointly control time and frequency resolution. The appropriate values of these parameters are determined empirically. The MST was applied to two-stage filtering for improving the signal-to-noise ratio in non-stationary and noisy time series. Results demonstrated better energy concentration in the time–frequency plane and effective removal of both background and localized noise. However, its performance decreases for highly noisy signals, indicating the need for further refinement.

3.2.3. Visual classification-oriented: Kondaveeti et al. [46] proposed a Modified ST (MST) variant focused on visual representation and classification rather than filtering. Their method enhances the time–frequency depiction of waveform distortions, allowing clearer visualization of transient and harmonic components. Case studies demonstrated that this version produces well-defined contours in the time–frequency plane, facilitating simple visual identification of distortion types and associated frequency components. The approach thus provides an effective diagnostic tool for analyzing and classifying waveform irregularities through intuitive visualization.

3.2.4. **Cross-MST**: Assous et al. **[47]** extended previous works by George et al. **[45]** and Kondaveeti et al. **[46]** to develop the Cross-MST, a variant designed to improve phase and synchronization estimation in noisy environments. By combining features from earlier MST formulations, this method enhances noise robustness and time–frequency resolution. Tests on both simulated and real signals showed that Cross-MST provides more accurate and stable phase estimation**,** achieving roughly double the resolution of the conventional ST. Future developments aim to benchmark this technique against other time–frequency methods for improved analysis of non-stationary synchronization phenomena**.**

3.2.5. Multi-parameter MST: To further extend the adaptability of the ST, Moukadem et al. [25] introduced a multi-parameter formulation that incorporates four adjustable parameters (m, p, k, r) within the Gaussian window. These parameters allow fine control of the window's shape and bandwidth, enabling the transform to adapt dynamically to the characteristics of non-stationary signals. Initially applied to biomedical signals, particularly heart sound detection, this version demonstrated superior time–frequency resolution compared to classical methods such as STFT, SPWVD, and previous ST variants [42]. Subsequent studies by the same authors [48] extended its application to a wide range of synthetic and real non-stationary signals, including modulated sinusoids, chirps, and short transients. The results confirmed improved localization, robustness to noise, and accurate estimation of instantaneous frequency. Beyond its proven performance, this multi-parameter approach stands out for its high flexibility and capacity for adaptation to signals exhibiting complex temporal and spectral variations. These characteristics make it particularly suitable for power quality analysis, where electrical signals often combine transient, harmonic, and modulated components. The multi-parameter MST therefore offers a promising framework for the development of optimized, adaptive tools in the study and diagnosis of power quality disturbances.

3.2.6. MST with parameters p, q: Zhang et al. [49] proposed a systematic parametric extension of the ST by introducing two independent parameters, p and q, that respectively vary linearly and exponentially with frequency. This dual dependence provides greater flexibility in adjusting the window width and controlling the time–frequency resolution of the transform. Applied to power quality analysis, this version of the MST demonstrated improved disturbance characterization and higher robustness to noise. Furthermore, it reduced feature redundancy, requiring only a few representative features from the time–frequency matrix for effective classification.

3.2.7. MST for SVM classification: Farida Hanim M. Noh et al. [50]applied the MST to feature extraction for the automatic classification of power quality disturbances using Support Vector Machines (SVM)**.** Their approach combined the formulation of

Mansinha et al. [36], where the parameter p is constant, with that of Assous et al. [30], who made p vary linearly with frequency to adapt the window function. Nineteen features were extracted from the time–frequency representation for classification. The method proved robust under both noisy and noise-free conditions, providing clear differentiation between disturbance types. However, the authors noted that broader validation on diverse operating conditions and comparison with other methods would be needed to fully assess its potential.

3.2.8. Double Resolution S-transform: To enhance resolution while lowering computational cost, Jianmin Li et al. [51]introduced the Double-Resolution S-Transform (DRST) combined with an optimized DAG-SVM classifier. This approach merges two MST with different window width parameters (λ_1, λ_2) allowing improved control of time–frequency resolution. The DRST achieves higher precision and reduced computational load**,** while the linear kernel DAG-SVM further minimizes processing and storage demands, enabling real-time implementation. Experimental results demonstrated strong classification accuracy and robustness, validated on hardware platforms. However, further research is needed to optimize parameters and assess dynamic performance under varying signal conditions.

3.2.9. Optimized ST: Building on the concept of dynamic parameter optimization, Qiu Tang et al. [52] developed the Optimized S-Transform (OST), which integrates adaptive adjustment of the window parameters with a Kernel Support Vector Machine (KSVM) classifier to improve robustness against mixed nonlinear disturbances. The OST maintains the same formulation as the DRST but introduces adaptive tuning to maximize energy concentration in the time–frequency plane. This enhances temporal resolution at low frequencies and frequency resolution at high frequencies, enabling precise characterization of complex disturbances. Combined with the KSVM, the method automatically extracts and learns discriminant time–frequency features, achieving better separability and stronger noise immunity than traditional approaches. Simulations and experimental validation confirmed the high accuracy and robustness of the OST-KSVM framework, particularly for mixed nonlinear disturbance detection.

3.2.10. Bi-Gaussian and Modified Incomplete S-transform: To further reduce computational complexity, Gong et al. [53] introduced a Bi-Gaussian S-Transform (BGST) and a Modified Incomplete S-Transform (MIST). The approach replaces the traditional Gaussian window with a bi-Gaussian function, allowing the transform to focus only on key frequency points, thereby lowering the computational load. Simulations on various power quality disturbances including voltage sags, swells, interruptions, and transients showed that MIST significantly decreases computation time while maintaining high detection accuracy. By tracking the maximum power spectrum dynamics, the method efficiently identifies disturbance start and end times, as well as amplitude variations. Overall, BGST and MIST provide a faster and more accurate alternative to the standard S-transform for practical disturbance detection.

3.2.11. Sigmoid ST: Fuyan Guo et al. [54]proposed the Sigmoid ST, which uses a sigmoid function to modulate the window width as a function of frequency. The parameters of the sigmoid controlling its slope and scale allow smooth, adaptive variation of the window, improving the readability of the time–frequency spectrum. Experimental analyses showed that this modulation yields a smoother S-matrix and enhances both temporal resolution at low frequencies and frequency resolution at high frequencies, making it particularly effective for power quality disturbance analysis. The authors also introduced Time Maximum Amplitude (TMA) and Frequency Maximum Amplitude (FMA) curves for intuitive feature extraction and suggested integrating neural networks for automated disturbance classification in future work.

3.2.12. **Improved generalized ST:** Chao Sun et al. [55] proposed an Improved GST (IGST) by introducing three additional parameters (λ, p, b) to refine low-frequency resolution. In this formulation, λ controls the time width and bandwidth, p adjusts the influence of frequency on λ, and b shifts the frequency coordinate position, enhancing the clarity of low-frequency components. This parametrization overcomes the limitation of the standard ST, where the Gaussian window is fixed, allowing adaptive control of time–frequency resolution. Applied to synthetic and real non-stationary seismic data, the IGST demonstrated improved signal-to-noise ratio and vertical resolution, confirming its effectiveness for analyzing signals with strong low-frequency content.

3.2.13. **DOST + CS – Hybrid approach:** Muhammad Abubakar et al. [56] proposed a hybrid approach combining the Discrete Orthogonal ST (DOST) [57] with Compressed Sampling (CS**)** to reduce data size while maintaining high classification accuracy. In this method, DOST is used for feature extraction, while CS compresses the data to lower computational requirements and facilitate real-time processing. Experimental results demonstrated that this hybrid technique achieves excellent classification performance, even under noisy conditions, confirming its suitability for power quality monitoring. Furthermore, its efficiency and adaptability make it promising for other applications such as image processing, hyperspectral imaging, and speech recognition.

3.2.14. **Summary of Optimized ST:** The optimized forms of the ST were developed to improve time–frequency resolution, noise robustness, and computational efficiency. They introduce adaptive parameters and optimization techniques to automatically adjust the analysis window to signal characteristics. Early improvements (IST, MST) focused on controlling window width, while later versions (Cross-MST, Multi-Parameter MST) added multiple parameters for greater flexibility and accuracy. Advanced methods integrated machine learning (SVM, KSVM) and optimization algorithms to enhance automatic feature extraction and disturbance classification. More recent approaches, such as BGST, MIST, IGST, and DOST+CS, aimed to reduce computational cost while maintaining precision, making the transform suitable for real-time applications.

Overall, these developments mark a shift toward adaptive and efficient versions of the ST. Among them, the multi-parameter approach by Moukadem et al.[25] stands out for its high flexibility and robustness, serving as the basis for the GAOA method proposed in this study.

4. The proposed method: GAOA

As shown in the previous section, several authors have proposed adaptations of the ST to improve its flexibility and ability to analyze complex signals. Among these approaches, the one proposed by Moukadem et al. [58]appears to be a solution particularly suited to our problem of detecting disturbances in electrical power quality.
One of the drawbacks of the ST is that the standard deviation is assigned uniformly to all signal components and frequencies. This means that the standard deviation is always defined as the reciprocal of the frequency[59]. However, there are situations where certain signals

could benefit from a different standard deviation value for the window function used in the ST. This limitation can affect the ST's ability to accurately represent the temporal and frequency characteristics of complex signals.

In the context of improving the ST, a simple but powerful approach is to introduce an adaptation of the standard deviation of the window function. This adaptation would allow for better adjustment to the specific properties of the signals being studied and could improve time and frequency resolution. Previous studies [55,59–66] have shown that this modification of the standard deviation can have a significant impact on the representation of signals with variable frequency components and short-duration transients.

By adjusting the standard deviation of the window function in the ST according to the characteristics of the signal, it is possible to obtain better separation of the different components of the signal in the time-frequency domain. This improvement would allow for more accurate analysis of complex signals and better resolution of the temporal and frequency structures present.

Modifying this standard deviation would allow control of the Gaussian window, which is done by introducing a new parameter into the standard deviation. Various authors have introduced a single adjustment parameter p, which allows control of the width of the Gaussian window in time- frequency analysis. This parameter acts directly on the standard deviation of the window function, and therefore on the trade-off between temporal resolution and frequency resolution. However, this approach has a significant limitation: the time-frequency resolution is entirely determined by this single parameter, which restricts the flexibility of the method. Conversely, introducing too many parameters (e.g., more than three) could significantly burden the optimization process by increasing computational complexity. A promising approach to overcoming this limitation is to explore more advanced optimization strategies that automatically find the optimal values for a set of tuning parameters. This could be achieved using multi-objective optimization techniques or machine learning algorithms, which would allow the parameter space to be searched efficiently and the best compromise between temporal and frequency resolution to be found. Such an approach would preserve the simplicity of the tuning process while offering increased flexibility to adapt the time-frequency resolution to the specific characteristics of the signals under study.

The approach proposed by Moukadem et al. [25] follows this approach by introducing an optimized ST with four adjustable parameters (m, p, k, r) that allow the standard deviation of the Gaussian window to be dynamically adjusted.

Proposals for ST, where the standard deviation is fixed or not enough parameters are introduced into the window, face implementation complexity and resource overloads, making them less suitable for many applications. The proposal by Moukadem et al. offers simplicity of implementation while providing effective time-frequency resolution. The introduction of four adjustable parameters offers greater flexibility to adapt the time-frequency resolution to the properties of the analyzed signal. This allows for better separation of signal components in the time-frequency domain and more accurate analysis of complex signals, particularly those with variable frequencies and short transients.

The effectiveness of this approach has been demonstrated in the context of detecting heart sound splitting and in the analysis of synthetic and real stationary signals[48]. This method uses a GA to automatically select the optimal parameters (m, p, k, r) based on a measure of energy concentration[43]. This optimization approach allows the best compromise between temporal and frequency resolution to be found, thus offering great flexibility in adapting the ST to the specific characteristics of the signals under study.

In the context of electrical power quality, disturbances such as flicker, transients, harmonics, or voltage dips have complex time-frequency signatures. The flexibility provided by the four parameters m, p, k, and r of Moukadem et al. makes it possible to better isolate and represent these phenomena in the time-frequency domain, facilitating their detection and classification. Four parameters m, p, k, and r are thus introduced into the ST, whose expression is given by the following relationship:

$$S_x^{m,p,k,r}(\tau,f)=\int_{-\infty}^{+\infty}\left[h(t)\frac{|f|^r}{(mf^p+k)\sqrt{2\pi}}\times e^{\frac{-(\tau-t)^2 f^{2r}}{2(mf^p+k)^2}}e^{-i2\pi ft}\right]dt \tag{9}$$

The choice of Gaussian window parameters is critical in time-frequency analysis. Empirical tuning of *m, p, k,* and *r* is inadequate due to the diversity of power quality disturbances, each with distinct time-frequency signatures. Moreover, the inherent trade-off between time and frequency resolution demands precision that manual adjustment cannot systematically ensure, particularly under noise or varying operating conditions.

To overcome these limitations, we employ a GA to automatically optimize the parameters according to the signal characteristics. GA, inspired by natural selection, is well suited for global optimization through selection, crossover, mutation, and replacement [67].

The fitness function is based on the energy concentration measure [43] applied to the MST to maximize energy localization in the time-frequency plane.

$$CM(m,p,k,r)=\frac{1}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|S_x^{m,p,k,r}(t,f)\right|dtdf} \tag{10}$$

The general algorithm combines the parameterized ST with a GA to automatically select the optimal parameters m, p, k, and r to improve the time-frequency representation of electrical signals. The steps are as follows:

a. Signal generation and preprocessing

- Generate or acquire the signal h(t).
- Define parameters m, p, k, and r as optimization variables.

b. Initial calculation of the parameterized Stockwell Transform

- Apply a fast Fourier transform (FFT) to the signal to obtain H[m].
- Calculate the FFT of the parameterized Gaussian window w (τ, t) to obtain W [n, m].
- Perform a frequency shift on H[m] to obtain H[m+n].
- Multiply H [m + n] by W [n, m] to obtain B [n, m].
- Apply an inverse Fourier transform (IFFT) to produce the time-frequency representation S [n, t].

c. Iteration over discrete frequencies

Repeat the calculation for all discrete frequencies of interest.

d. Definition of the fitness function

Use an energy concentration measure to evaluate the quality of the representation obtained.

e. Optimization using a GA

- Initialization: Randomly generate a population of combinations of the parameters m, p, k, and r.
- Evaluation: Calculate the fitness value for each individual.
- Selection: Retain the best individuals.
- Crossover and mutation: Create new individuals to explore the search space.
- Optimization loop: Repeat the evaluation, selection, crossover, and mutation steps until the termination criterion is satisfied.

f. Final selection and application

- Select the individual with the best fitness value.
- Apply the optimized Stockwell Transform with these parameters to the signal *h(t)* to obtain

the final representation.

At the end of this process, the optimal parameters obtained are applied to the Stockwell Transform to provide a refined time-frequency representation, suitable for detecting and characterizing disturbances in electrical power quality.

5. Conclusion

This article presents an evolution of the ST towards an optimized version, introducing an adaptation of the standard deviation of the window function. This modification allows the ST to be better adjusted to the specific characteristics of the signals under study, thus improving time and frequency resolution and enabling a more accurate representation of the temporal and frequency characteristics of complex signals. The article highlights the importance of carefully selecting the parameters of the Gaussian window for effective time-frequency analysis. The use of a GA to automatically select the optimal parameters m, p, k, and r offers a promising approach to finding the best compromise between temporal and frequency resolution. By using the energy concentration measure as a fitness function, the GA allows the parameter space to be explored efficiently and global optimal solutions to be obtained. This approach offers increased flexibility to adapt the method to the analysis of power quality disturbances, while maintaining the simplicity of the tuning process. Previous work on the optimized ST has shown its potential for improving the representation of complex signals, particularly those with variable frequency components and short-duration transients. The approach proposed in this article represents a significant advance in this field, offering new prospects for more accurate analysis of complex signals and better resolution of the temporal and frequency structures present. The AOAG presented in this article, combining the adaptation of the standard deviation of the window function and the use of a GA to optimize the choice of optimal parameter values, is a promising method for improved time-frequency analysis. This approach opens up many prospects for a more detailed and accurate analysis of complex signals, particularly in the field of electrical power quality. It offers significant potential for improving the detection, diagnosis, and monitoring of disturbances on electrical networks, as well as for new applications in other areas of complex signals.

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