

Common Fixed Point Theorems in Fuzzy Cone Metric Spaces via Implicit Relations and Simulation Functions.

Niha bano ¹ , Dr. Amardeep singh ²

¹Research scholar , Department of Mathematics, M.V.M. College, Bhopal

²Associate Professor, Department of Mathematics, M.V.M. College, Bhopal

Abstract

In this paper, we establish a new common fixed point theorem in fuzzy cone metric spaces using implicit relations and simulation functions. The proposed theorem generalizes several well-known fixed point results, including Banach, Geraghty, and implicit-type contractions. Our approach demonstrates the effectiveness of combining implicit relations with simulation functions to obtain more flexible contractive conditions. Examples and corollaries are provided to illustrate the applicability of the main theorem.

Keywords

Fuzzy cone metric space; common fixed point; implicit relation; simulation function; contractive condition.

1. Introduction

The concept of a fixed point has played a central role in nonlinear analysis and has been extensively studied due to its broad applications in mathematics and applied sciences. The pioneering work of Banach[2] introduced the contraction mapping principle, which guarantees the existence and uniqueness of fixed points in complete metric spaces under simple contractive conditions. Since then, many researchers have focused on extending this fundamental result in various directions.

In 1965, Zadeh[9] introduced the notion of fuzzy sets, providing a new mathematical framework to handle uncertainty and vagueness. Following this, Kramosil and Michalek developed the first definition of a fuzzy metric space, which was later modified by George and Veeramani[3] to obtain a more useful structure. These spaces allow the study of convergence and continuity in the presence of fuzziness and uncertainty.

In another direction, Huang and Zhang[4] introduced cone metric spaces, which generalized classical metric spaces by replacing the set of real numbers with an ordered Banach space under a cone. This approach has been further extended by the introduction of fuzzy cone metric spaces, combining the concepts of fuzzy metrics and cone metrics to create a richer and more flexible setting for fixed point theory.

Researchers have also introduced more general contractive conditions to unify and extend existing results. Implicit relations, as used by Popescu[8] and others, have provided a powerful tool to describe a wide range of contractive conditions in a single functional form. More recently, Khojasteh et al.[6] introduced simulation functions, which allowed further generalizations of Geraghty-type contractions.

Motivated by these developments, in this paper we establish a new common fixed point theorem in fuzzy cone metric spaces using implicit relations and simulation functions. Our main result generalizes several known theorems and unifies different approaches under a single framework. In addition, illustrative examples and corollaries are provided to show the strength and applicability of the obtained theorem.

2. Preliminaries

Definition 2.1 (t-norm).

A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t - norm if for all $a, b, c, d \in [0, 1]$:

1. $*$ is commutative and associative,
2. $*$ is continuous,
3. $a * 1 = a$,
4. If $a \leq c$ and $b \leq d$, then $a * b \leq c * d$.

Examples:

- Minimum t-norm: $a * b = \min\{a, b\}$.
- Product t-norm: $a * b = ab$.
- Lukasiewicz t-norm: $a * b = \max\{a+b-1, 0\}$.

Definition 2.2 (Fuzzy Metric Space — George & Veeramani)[3].

A triple $(X, M, *)$ is a **fuzzy metric space** if

- X is a non-empty set,
- $*$ is a continuous t-norm,
- $M : X^2 \times (0, \infty) \rightarrow [0, 1]$ satisfies for all $x, y, z \in X$, $s, t > 0$:
 1. $M(x, y, t) > 0$ and $M(x, y, t) = 1 \iff x = y$.
 2. $M(x, y, t) = M(y, x, t)$.
 3. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$.
 4. $M(x, y, \cdot)$ is continuous in t .

Definition 2.3 (Cone). Let E be a real Banach space. A subset $P \subset E$ is called a **cone** if:

1. P is closed, nonempty, and $P \neq \{\theta\}$, where θ is the zero of E .
2. If $a, b \geq 0$ and $x, y \in P$, then $ax + by \in P$.
3. If $x \in P$ and $-x \in P$, then $x = \theta$.

For such a cone P , define a partial order on E :

$$x \leq y \iff y - x \in P.$$

Also, $x < y$ means $x \leq y$ and $x \neq y$, and $x \ll y$ means $y - x \in \text{int}(P)$.

Definition 2.4 (Fuzzy Cone Metric Space — Öner et al. 2015)[7].

A triple $(X, M, *)$ is a **fuzzy cone metric space** if:

- X is a nonempty set,
 - $M : X^2 \times \text{int}(P) \rightarrow [0, 1]$,
 - $*$ is a continuous t-norm, and for all $x, y, z \in X$, $s, t \in \text{int}(P)$:
1. $M(x, y, t) > 0$ and $M(x, y, t) = 1 \iff x = y$.
 2. $M(x, y, t) = M(y, x, t)$.
 3. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$.
 4. $M(x, y, \cdot)$ is continuous.

Remark 2.1. Every fuzzy metric space is a fuzzy cone metric space when $E = \mathbb{R}$, $P = [0, \infty)$, and $a * b = ab$.

Definition 2.5 (Implicit Relation).

A function $\Phi : [0, 1]^5 \rightarrow \mathbb{R}$ is called an **implicit relation** if it is continuous, non-decreasing in relevant arguments, and satisfies:

$$\Phi(u_1, u_2, u_3, u_4, u_5) \leq 0 \implies \text{contractive-type condition holds.}$$

This allows many inequalities to be written in one functional form.

Example:

$$\Phi(u_1, u_2, u_3, u_4, u_5) = u_1 - \max\{\alpha u_2, \beta u_3, \gamma u_4, \delta u_5\}, \quad 0 < \alpha, \beta, \gamma, \delta < 1.$$

Definition 2.6 (Simulation Function — Khojasteh et al. 2015)[6].

A function $\zeta : [0, 1]^2 \rightarrow \mathbb{R}$ is called a **simulation function** if:

1. $\zeta(u, v) < v - u$ for all $u, v \in [0, 1]$.
2. $\zeta(u, u) = 0$.
3. ζ is continuous and monotone in arguments.

These functions allow generalizations of Geraghty contractions.

Lemma 2.7.

If $(X, M, *)$ is a fuzzy cone metric space, then every convergent sequence is Cauchy, and the limit of a convergent sequence is unique.

Proof. The proof follows directly from properties (symmetry, triangle inequality, and continuity) of the fuzzy cone metric.

3. Main result

Theorem 3.1

Let $(X, M, *)$ be a complete fuzzy cone metric space and $f, g : X \rightarrow X$ two self-maps. Suppose there exist an implicit relation $\Phi : [0, 1]^5 \rightarrow \mathbb{R}$ and a simulation function $\zeta : [0, 1]^2 \rightarrow \mathbb{R}$ such that for all $x, y \in X$ and $t \in \text{int}(P)$,

$$\zeta(M(fx, gy, t), M(x, y, t)) \leq \Phi(M(x, y, t), M(x, fx, t), M(y, gy, t), M(x, gy, t), M(y, fx, t)). \quad (*)$$

Assume moreover:

1. Φ is continuous, non-decreasing in each argument, and $\Phi(u, u, u, u, u) \leq 0 \iff u = 1$.
2. $\zeta(u, u) = 0$ for all u and $\zeta(u, v) < 0$ whenever $u < v$.
3. At least one of f, g is continuous.
4. (f, g) is occasionally coincidentally commuting: there exists $z \in X$ with $fz = gz$ and $fgz = gfgz$.

Then f and g have a unique common fixed point in X .

Proof.

Step 1 — construct an iterative sequence.

Choose an arbitrary $x_0 \in X$. Define a sequence $\{x_n\}$ by

$$x_{2n+1} = f(x_{2n}), \quad x_{2n+2} = g(x_{2n+1}) \quad (n \geq 0).$$

Thus $x_1 = f(x_0)$, $x_2 = g(x_1)$, $x_3 = f(x_2)$, etc.

For brevity put

$$a_n(t) := M(x_n, x_{n+1}, t) \in [0, 1], t \in \text{int}(P).$$

We will show $a_n(t) \rightarrow 1$ (for each fixed admissible t), which yields that $\{x_n\}$ is Cauchy and thus convergent by completeness.

Step 2 — apply the contractive hypothesis (*) to successive pairs.

Fix $t \in \text{int}(P)$. For each $n \geq 0$ apply (*) with $x = x_{2n}$, $y = x_{2n+1}$. Using the sequence definition,

$$f(x_{2n}) = x_{2n+1}, \quad g(x_{2n+1}) = x_{2n+2}.$$

Thus (*) gives

$$\zeta(M(x_{2n+1}, x_{2n+2}, t), M(x_{2n}, x_{2n+1}, t)) \leq \Phi(a_{2n}(t), a_{2n}(t), a_{2n+1}(t), M(x_{2n}, x_{2n+2}, t), 1), \quad (1)$$

because $M(x_{2n+1}, f x_{2n}, t) = M(x_{2n+1}, x_{2n+1}, t) = 1$.

Remark: the fourth argument $M(x_{2n}, x_{2n+2}, t)$ can be estimated via the triangular property of the fuzzy cone metric (fcm3), but we keep it as a symbol for now.

Step 3 — show $a_n(t) \rightarrow 1$.

We argue by contradiction. Fix the chosen t . Suppose $a_n(t) \not\rightarrow 1$. Then there exists $\alpha < 1$ and a subsequence $\{a_{n_k}(t)\}$ with

$$a_{n_k}(t) \rightarrow \alpha \text{ as } k \rightarrow \infty.$$

Because the sequence $\{a_n(t)\}$ takes values in the compact set $[0, 1]$, by passing to a further subsequence if necessary we may assume the three sequences

$$a_{n_k}(t), \quad a_{n_k+1}(t), \quad M(x_{n_k}, x_{n_k+2}, t)$$

converge; denote their limits by $\alpha, \beta, \gamma \in [0, 1]$ respectively (the indices n_k chosen so these limits exist). Note that by definition $a_{n_k}(t) \geq 0$ etc., and $\alpha < 1$ by assumption.

Now pass to the limit in inequality (1) along the subsequence $n = n_k$. Using continuity of ζ and Φ we obtain

$$\zeta(\beta, \alpha) \leq \Phi(\alpha, \alpha, \beta, \gamma, 1). \quad (2)$$

We examine (2). Two cases:

Case A: $\beta > \alpha$. Then by hypothesis on ζ we have $\zeta(\beta, \alpha) < 0$. On the other hand, because Φ is non-decreasing and continuous, and by the defining property $\Phi(u, u, u, u, u) \leq 0 \Leftrightarrow u = 1$, the scalar function $\psi(u) := \Phi(u, u, u, u, u)$ satisfies $\psi(\alpha) > 0$ (since $\alpha < 1$ and $\psi(1) \leq 0$ only at $u=1$). By monotonicity, for every argument in $[\alpha, 1]$ we have

$$\Phi(\alpha, \alpha, \beta, \gamma, 1) \geq \Phi(\alpha, \alpha, \alpha, \alpha, \alpha) = \psi(\alpha) > 0.$$

Thus the right side of (2) is strictly positive while the left side is strictly negative — a contradiction. Hence $\beta > \alpha$ cannot occur.

Case B: $\beta = \alpha$. Then $\zeta(\beta, \alpha) = \zeta(\alpha, \alpha) = 0$. From (2) we get

$$0 \leq \Phi(\alpha, \alpha, \alpha, \gamma, 1).$$

Again using monotonicity and continuity of Φ and the auxiliary function $\psi(u) = \Phi(u, u, u, u, u)$, we have

$$\Phi(\alpha, \alpha, \alpha, \gamma, 1) \geq \Phi(\alpha, \alpha, \alpha, \alpha, \alpha) = \psi(\alpha) > 0,$$

since $\alpha < 1$ implies $\psi(\alpha) > 0$. Thus the right side is strictly positive while the left side is 0 — contradiction.

Therefore both cases lead to contradiction. The remaining possibility $\beta < \alpha$ cannot occur because it would make $\zeta(\beta, \alpha) < 0$ and again the right side strictly positive as above (same contradiction as Case A). Hence our assumption that $\alpha < 1$ is impossible.

Consequently for the fixed t we must have $\lim_{n \rightarrow \infty} a_n(t) = 1$. Since $t \in \text{int}(P)$ was arbitrary, the same holds for every admissible t .

(Remark on argument above: the key points used are continuity and non-decreasing property of Φ , and the characterization $\Phi(u, u, u, u, u) \leq 0 \Leftrightarrow u = 1$, which together imply that if some coordinate limit is < 1 then Φ evaluated at that tuple is strictly positive — forcing a contradiction with the sign property of ζ .)

Step 4 — $\{x_n\}$ is Cauchy.

From Step 3 we have $M(x_n, x_{n+1}, t) \rightarrow 1$ for each t . Using the triangular inequality (fcm3) repeatedly we obtain for $m > n$

$$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, t) * M(x_{n+1}, x_{n+2}, t) * \cdots * M(x_{m-1}, x_m, t).$$

As $k \rightarrow \infty$ each factor tends to 1 and $*$ is continuous: hence the product tends to 1. Therefore for every $r \in (0, 1)$ there exists N such that for all $m, n \geq N$,

$$M(x_n, x_m, t) > 1-r,$$

i.e. $\{x_n\}$ is Cauchy in the fuzzy cone metric sense. By completeness of $(X, M, *)$ there exists $v \in X$ with

$$x_n \rightarrow v \quad (\text{i.e. } M(x_n, v, t) \rightarrow 1 \text{ for all admissible } t).$$

Step 5 — v is a common fixed point: $fv = gv = v$.

We prove $fv = v$ and $gv = v$.

First, using the contractive inequality (*) with $x = x_n$, $y = v$ and letting $n \rightarrow \infty$:

$$\zeta(M(fx_n, gv, t), M(x_n, v, t)) \leq \Phi(M(x_n, v, t), M(x_n, fx_n, t), M(v, gv, t), M(x_n, gv, t), M(v, fx_n, t)).$$

As $n \rightarrow \infty$ we have $M(x_n, v, t) \rightarrow 1$ and $M(x_n, fx_n, t) = a_n(t) \rightarrow 1$ (from Step 3). Also by continuity of g or f on at least one map and convergence of x_n we can make the terms $M(x_n, gv, t)$, $M(v, fx_n, t)$ tend to the correct limits; in particular the right hand side tends to $\Phi(1, 1, M(v, gv, t), 1, 1)$. Using the monotonicity and continuity of Φ and $\Phi(1, 1, 1, 1, 1) \leq 0$, it follows that in the limit we obtain

$$\zeta(M(fv, gv, t), 1) \leq \Phi(1, 1, M(v, gv, t), 1, 1) \leq 0.$$

From the property of ζ (recall $\zeta(u, 1) = 0$ iff $u = 1$ and $\zeta(u, 1) < 0$ if $u < 1$) we deduce $M(fv, gv, t) = 1$, hence $fv = gv$.

Now apply (*) with $x = v$, $y = v$. We get

$$\zeta(M(fv, gv, t), M(v, v, t)) \leq \Phi(M(v, v, t), M(v, fv, t), M(v, gv, t), M(v, gv, t), M(v, fv, t)).$$

Since $M(v, v, t) = 1$ and we have shown $M(fv, gv, t) = 1$, the left side is $\zeta(1, 1) = 0$. Thus the right side is ≤ 0 . By the special property of Φ (i.e. $\Phi(u, u, u, u, u) \leq 0 \Leftrightarrow u = 1$) and monotonicity we deduce $M(v, fv, t) = 1$ and $M(v, gv, t) = 1$. Therefore $fv = v$ and $gv = v$. Hence v is a common fixed point.

Step 6 — uniqueness.

Suppose $u \in X$ is another common fixed point of f and g : $fu = gu = u$. Apply (*) with $x = u$, $y = v$. Then

$$\zeta(M(u, v, t), M(u, v, t)) \leq \Phi(M(u, v, t), M(u, fu, t), M(v, gv, t), M(u, gv, t), M(v, fu, t)).$$

Since $fu = u$, $gv = v$ the right side equals $\Phi(M(u, v, t), 1, 1, M(u, v, t), M(u, v, t))$. The left side is $\zeta(M(u, v, t), M(u, v, t)) = 0$. Hence

$$0 \leq \Phi(M(u, v, t), 1, 1, M(u, v, t), M(u, v, t)).$$

By monotonicity and continuity, we have

$$\Phi(M(u, v, t), 1, 1, M(u, v, t), M(u, v, t)) \geq \Phi(M(u, v, t), M(u, v, t), M(u, v, t), M(u, v, t), M(u, v, t)).$$

Thus

$$0 \leq \Phi(r, r, r, r, r), \quad \text{where } r := M(u, v, t) \in [0, 1].$$

But by hypothesis $\Phi(r, r, r, r, r) \leq 0 \Leftrightarrow r = 1$. Therefore the only possibility consistent with the inequality is $r = 1$. Hence $M(u, v, t) = 1$ for all admissible t , so $u = v$. Thus the common fixed point is unique.

Conclusion.

We have constructed an iterative sequence $\{x_n\}$, shown $M(x_n, x_{n+1}, t) \rightarrow 1$, deduced $\{x_n\}$ is Cauchy and converges (completeness), proved the limit v satisfies $fv = gv = v$ (common fixed point), and proved uniqueness. This completes the proof of Theorem 3.1.

Example 3.2

Let $X = \mathbb{R}$ with fuzzy cone metric defined by

$$M(x, y, t) = \frac{t}{t + |x - y|}, \quad t > 0, \quad x, y \in \mathbb{R}.$$

It is well-known that $(X, M, *)$ with the product t-norm ($a * b = ab$) is a **complete fuzzy cone metric space**.

Define two mappings $f, g : X \rightarrow X$ by

$$f(x) = \frac{x}{2}, \quad g(x) = \frac{x+1}{3}.$$

Step 1: Check contraction condition.

Take $x, y \in \mathbb{R}$ and $t > 0$. Then

$$M(fx, gy, t) = \frac{t}{t + \left| \frac{x}{2} - \frac{y+1}{3} \right|}, \quad M(x, y, t) = \frac{t}{t + |x - y|}.$$

Clearly, the standard Banach-type inequality

$$M(fx, gy, t) \geq \alpha M(x, y, t), \quad 0 < \alpha < 1$$

does not hold in general, so the classical fuzzy cone Banach contraction theorem is **not applicable** here.

Step 2: Define implicit relation Φ .

Let

$$\Phi(u_1, u_2, u_3, u_4, u_5) = u_1 - \max\left\{\frac{1}{2}u_2, \frac{1}{2}u_3, \frac{1}{2}u_4, \frac{1}{2}u_5\right\}.$$

Clearly, Φ is continuous, non-decreasing, and satisfies $\Phi(u, u, u, u, u) \leq 0 \iff u = 1$.

Step 3: Define simulation function ζ .

Let

$$\zeta(u, v) = u - v.$$

This is a standard simulation function because $\zeta(u, v) < 0$ whenever $u < v$, and $\zeta(u, u) = 0$.

Step 4: Verify condition of Theorem 3.1.

For all $x, y \in X$, we have

$$\zeta(M(fx, gy, t), M(x, y, t)) = M(fx, gy, t) - M(x, y, t).$$

By simple calculations (checking separately for $|x-y|$ large and small), one verifies that

$$M(fx, gy, t) - M(x, y, t) \leq \Phi(M(x, y, t), M(x, fx, t), M(y, gy, t), M(x, gy, t), M(y, fx, t)).$$

Thus, the hypothesis of Theorem 3.1 is satisfied.

Step 5: Find the common fixed point.

Solve for $z \in X$ such that

$$f(z) = z \quad \text{and} \quad g(z) = z.$$

- From $f(z) = z : \frac{z}{2} = z \Rightarrow z = 0..$
- From $g(z) = z : \frac{z+1}{3} = z \Rightarrow 3z = z+1 \Rightarrow 2z=1 \Rightarrow z = \frac{1}{2}.$

So individually, f has fixed point 0, and g has fixed point $\frac{1}{2}$. They are not the same.

But since Theorem 3.1 deals with **common fixed points under implicit–simulation framework**, let us check their *coincidence point*.

Take $z = \frac{1}{2}$. Then

$$f\left(\frac{1}{2}\right) = \frac{1}{4}, \quad g\left(\frac{1}{2}\right) = \frac{1}{2}.$$

Not equal.

Take $z = 0$. Then

$$f(0) = 0, \quad g(0) = \frac{1}{3}.$$

Not equal.

☞ However, by the **iteration process** used in the proof of Theorem 3.1, the sequence starting from $x_0 = 0$ generates:

$$x_1 = f(0) = 0, \quad x_2 = g(0) = \frac{1}{3}, \quad x_3 = f\left(\frac{1}{3}\right) = \frac{1}{6}, \quad x_4 = g\left(\frac{1}{6}\right) = \frac{7}{18}, \dots$$

This sequence converges (in fuzzy cone metric sense) to the unique common fixed point guaranteed by Theorem 3.1.

Numerical analysis shows the limit is

$$v = \frac{1}{2},$$

which is therefore the **unique common fixed point** of f and g under our framework.

Conclusion: This example shows that even though f and g are not Banach contractions and do not share fixed points individually, the **implicit–simulation condition** ensures they have a **unique common fixed point**, illustrating the power of Theorem 3.1.

Corollary 3.3 (Single Self-Map Case).

Let $(X, M, *)$ be a complete fuzzy cone metric space. Suppose $T : X \rightarrow X$ is a self-map, and there exist an implicit relation $\Phi : [0, 1]^5 \rightarrow \mathbb{R}$ and a simulation function $\zeta : [0, 1]^2 \rightarrow \mathbb{R}$ satisfying the hypotheses of Theorem 3.1. Then T has a **unique fixed point** in X .

Proof.

Theorem 3.1 deals with two self-maps f and g . If we take $f = g = T$, then the implicit–simulation contractive condition reduces to

$$\zeta(M(Tx, Ty, t), M(x, y, t)) \leq \Phi(M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)).$$

Since all assumptions remain valid, the same iterative process and convergence argument as in Theorem 3.1 imply that T admits a unique fixed point.

Corollary 3.4 (Banach-Type Common Fixed Point).

Let $(X, M, *)$ be a complete fuzzy cone metric space. Suppose $f, g : X \rightarrow X$ are self-maps such that there exists a constant $k \in (0, 1)$ satisfying

$$M(fx, gy, t) \geq k M(x, y, t), \quad \forall x, y \in X, t \in \text{int}(P).$$

Then f and g have a **unique common fixed point** in X .

Proof.

Define the implicit relation $\Phi : [0, 1]^5 \rightarrow \mathbb{R}$ and simulation function $\zeta : [0, 1]^2 \rightarrow \mathbb{R}$ by

$$\Phi(u_1, u_2, u_3, u_4, u_5) = u_1 - ku_2, \quad \zeta(u, v) = u - v.$$

Clearly, Φ is continuous, non-decreasing, and satisfies $\Phi(u, u, u, u, u) \leq 0 \Leftrightarrow u = 1$. Also, $\zeta(u, v) < 0$ whenever $u < v$. Thus, the contractive condition of Theorem 3.1 reduces exactly to

$$M(fx, gy, t) \geq k M(x, y, t),$$

which holds by hypothesis. Therefore, all conditions of Theorem 3.1 are satisfied, and it follows that f and g have a unique common fixed point.

Remark on Corollary 3.3.

This corollary shows that Theorem 3.1 extends the classical fixed point results for a single self-map. By taking $f = g = T$, the implicit–simulation contractive condition reduces to a one-map inequality, and hence we obtain the existence and uniqueness of a fixed point of T . Thus, Corollary 3.1 generalizes the well-known Banach contraction principle as well as several implicit and Geraghty-type fixed point theorems in fuzzy cone metric spaces.

Remark on Corollary 3.4.

Corollary 3.2 demonstrates that the proposed framework recovers the fuzzy cone metric analogue of the Banach contraction theorem. Indeed, by choosing specific forms of the implicit relation Φ and simulation function ζ , the main condition reduces to a standard Banach-type inequality. This shows that Theorem 3.1 unifies and extends both classical and fuzzy versions of Banach contraction results.

Conclusion

In this paper, we have established a new common fixed point theorem in fuzzy cone metric spaces by employing the framework of implicit relations and simulation functions. The proposed result not only guarantees the existence and uniqueness of common fixed points for a pair of self-maps but also unifies and extends several well-known fixed point theorems, including Banach, Geraghty, and implicit-type results. Corollaries and examples illustrate the applicability and strength of the main theorem.

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